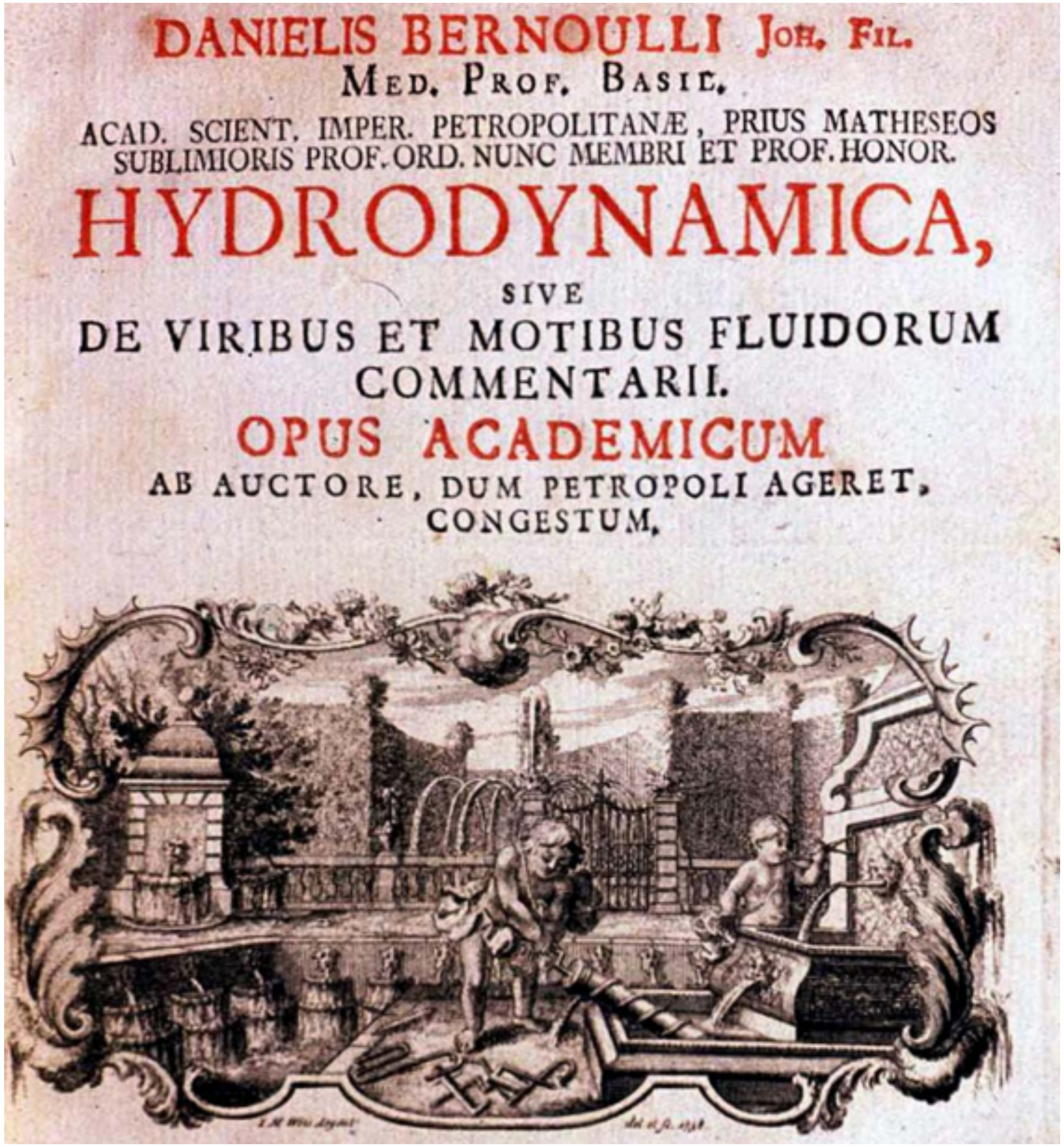
Hydrodynamics and quantum anomalies

Dam Thanh Son (University of Chicago)
EFI Colloquium (April 25, 2016)

Plan of the talk

- Hydrodynamics
- Anomalies
- Gauge/gravity duality
- Hydrodynamics with anomalies
- (a simplified version of a complex story)



DANIELIS BERNOULLI JOH. FIL.
MED. PROF. BASIL.
ACAD. SCIENT. IMPER. PETROPOLITANÆ, PRIUS MATHESEOS SUBLIMIORIS PROF. ORD. NUNC MEMBRI ET PROF. HONOR.

HYDRODYNAMICA,

SIVE
DE VIRIBUS ET MOTIBUS FLUIDORUM COMMENTARII.

OPUS ACADEMICUM
AB AUCTORE, DUM PETROPOLI AGERET, CONGESTUM.

Bernoulli, "Hydrodynamica", 1738

Navier-Stokes equation

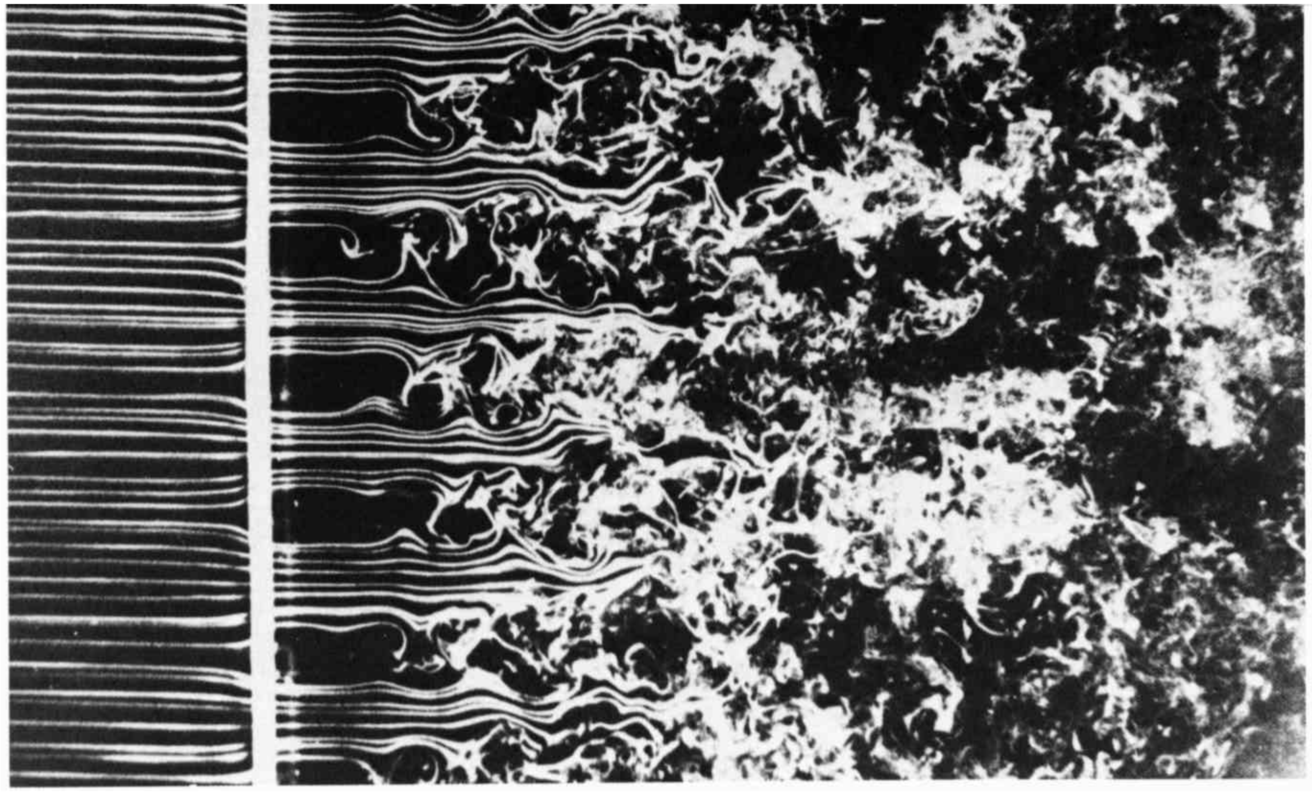
- Continuity equation $$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \mathbf{v}) = 0$$

Momentum conservation $$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{v} = -\frac{1}{\rho}\boldsymbol{\nabla} P + \nu \nabla^2 \mathbf{v}$$

(+ Energy conservation)

This simple set of equations lead to many complex physical phenomena

Hydrodynamic phenomena



(Van Dyke, Album of fluid motion)

Relativistic hydrodynamics

- The conservation of energy, momentum, baryon charge

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu j^\mu = 0$$

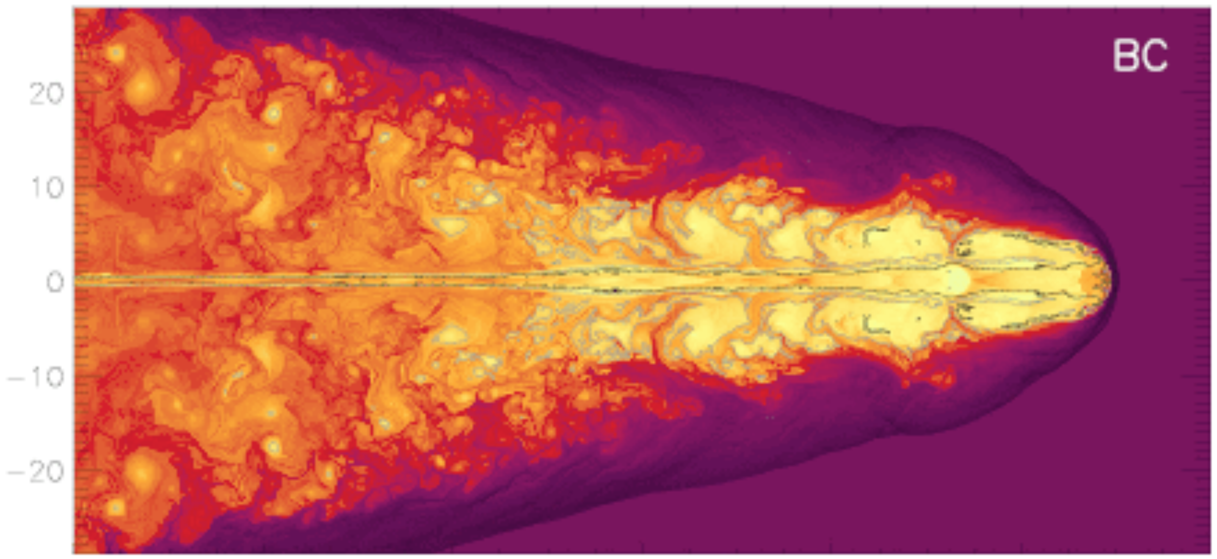
"Constitutive relations"

$$\vec{j} = \rho\vec{v} - D\vec{\nabla}\rho$$

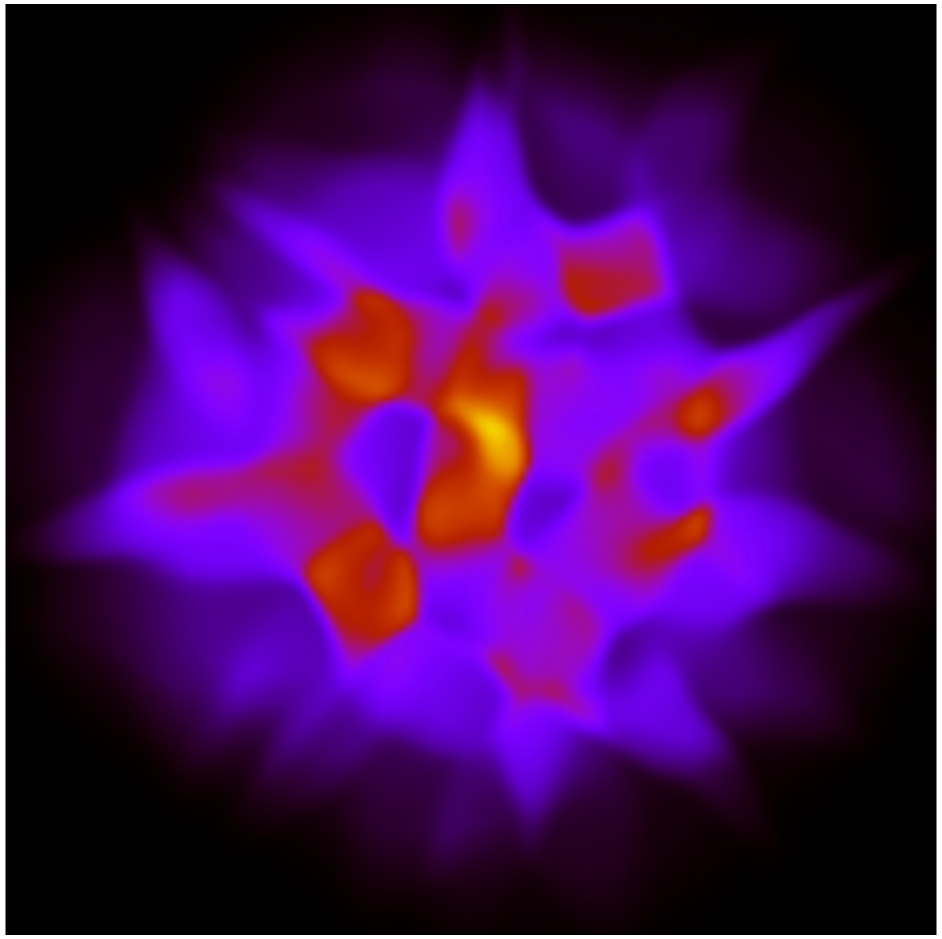
advection diffusion

a similar equation for $T^{\mu\nu}$

Applications of relativistic hydrodynamics



astrophysics
(astro-ph/0111369)



heavy ion collisions
(Bjorn Schenke)

Hydro: a classical theory

- Classical fluid even when constituents are quantum (quark gluon plasma for example): described by the same equations
- Quantum effects disappear when coarse grained over a large box
- Soft bosonic modes are classical

$$n(\omega) = \frac{1}{e^{\omega/T} - 1} \approx \frac{T}{\omega}$$

Two path to quantum hydrodynamics

- Path 1: started with a letter from Kapitza to Molotov (Prime Minister of the USSR) dated April 6, 1939

Recently, while studying helium near absolute zero, I found a whole series of new phenomena, possibly shedding light on a most mysterious area of modern physics... I need a theorist's help... In the Soviet Union the theorist who knows the subject completely and whom I need is Landau, but he has been under arrest for a year...

Can you ask NKVD to speed up Landau's case? If not, can we use his brain for scientific work while he is sitting in his cell?...

Landau 1941

- Landau, later Khalatnikov and others, constructed a hydrodynamic theory of superfluid helium
- Different from usual hydrodynamics because of spontaneous symmetry breaking due to Bose condensation
- SSB gives rise to a new hydrodynamic mode (superfluid velocity)

Path 2: anomalies

- Very recently (2007-now): quantum effects discovered in hydrodynamics of normal fluids
  - related to quantum anomalies
- It was almost an accidental discovery, made through gauge-gravity duality

Beginning: π^0 decay

PHYSICAL REVIEW VOLUME 76, NUMBER 8 OCTOBER 15, 1949

On the Use of Subtraction Fields and the Lifetimes of Some Types of Meson Decay

J. STEINBERGER*
The Institute for Advanced Study, Princeton, New Jersey
(Received June 13, 1949)

"...The method [Pauli-Villars] is, however, not without ambiguity......"

Progress of Theoretical Physics Vol. IV, No. 3, July~Sept., 1949.

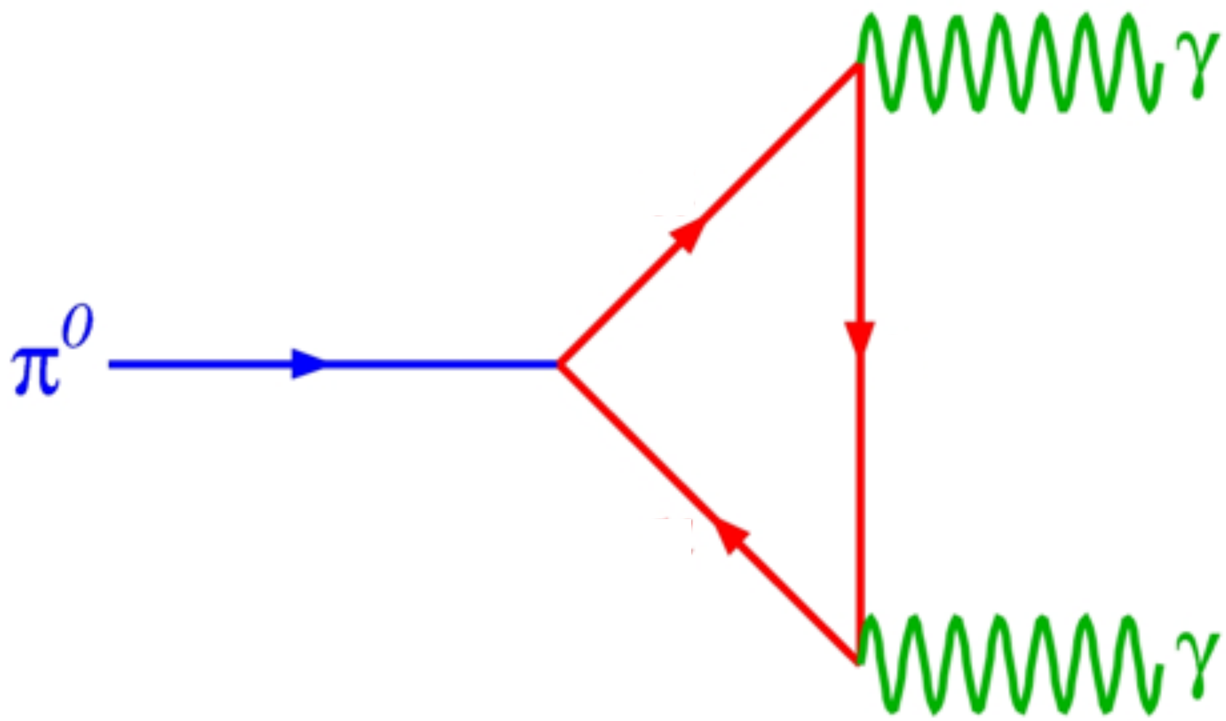
On the γ-Decay of Neutral Meson.

Hiroshi FUKUDA and Yoneji MIYAMOTO.
Physics Institute, Tokyo University.

(Received May 16, 1949)

By using the method of evaluation which has been applied by Schwinger…we have obtained the convergent but non-gauge covariant result for the γ decay of neutral meson….Thus, in the present state of the field theory, we cannot give an unambiguous life-time for neutral meson.

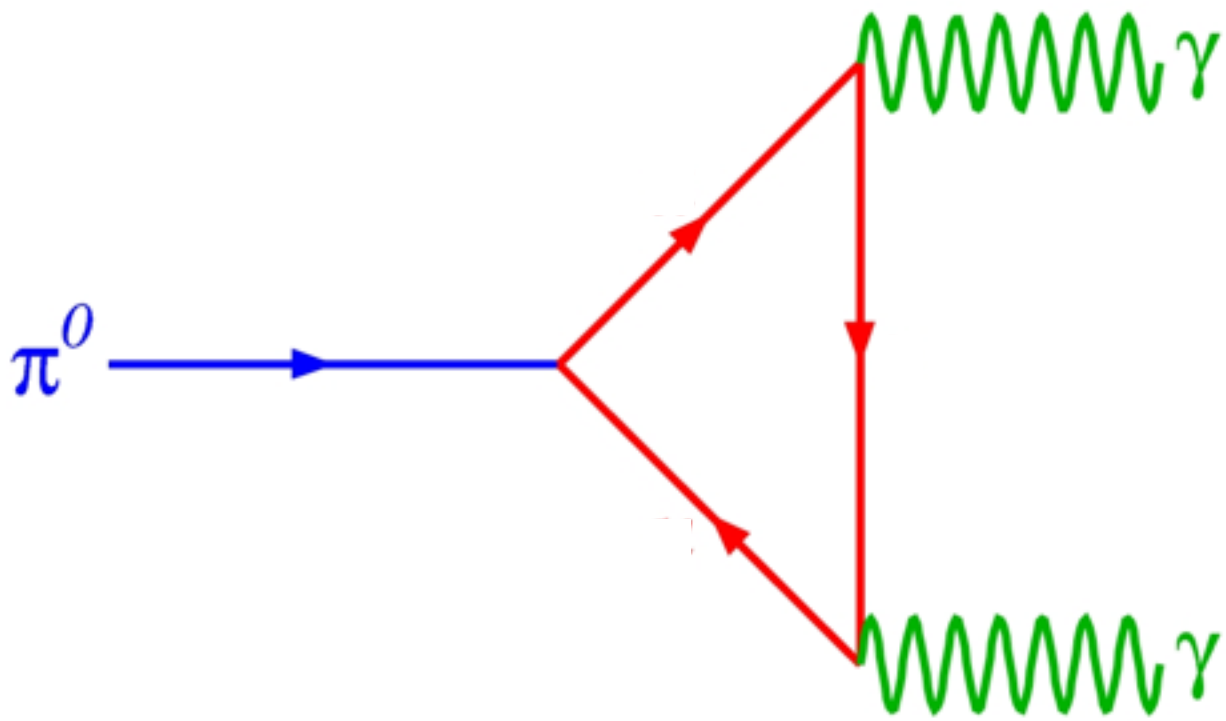
Puzzle of π^0 decay



Conditionally convergent int
Result depends on the way UV cutoff is imposed

$$\int_{-\infty}^{\infty} dx\,[f(x+a)-f(x)] = \int_{-\infty}^{\infty} dx\,[af'(x)+O(a^2)] = a[f(\infty)-f(-\infty)]$$

Puzzle of π⁰ decay



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$$\int_{-\infty}^{\infty} dx\,[f(x+a)-f(x)] = \int_{-\infty}^{\infty} dx\,[af'(x)+O(a^2)] = a[f(\infty)-f(-\infty)]$$

$$\Big\downarrow\; x \to x-a$$

$$\int_{-\infty}^{\infty} dx\,[f(x)-f(x)] = 0$$

Anomaly

- The key to the understanding of the pion decay puzzle was identified by Adler, Bell, Jackiw (1969)
- In massless electrodynamics, numbers of left- and right-handed electrons are not conserved separately in quantum theory

Chirality

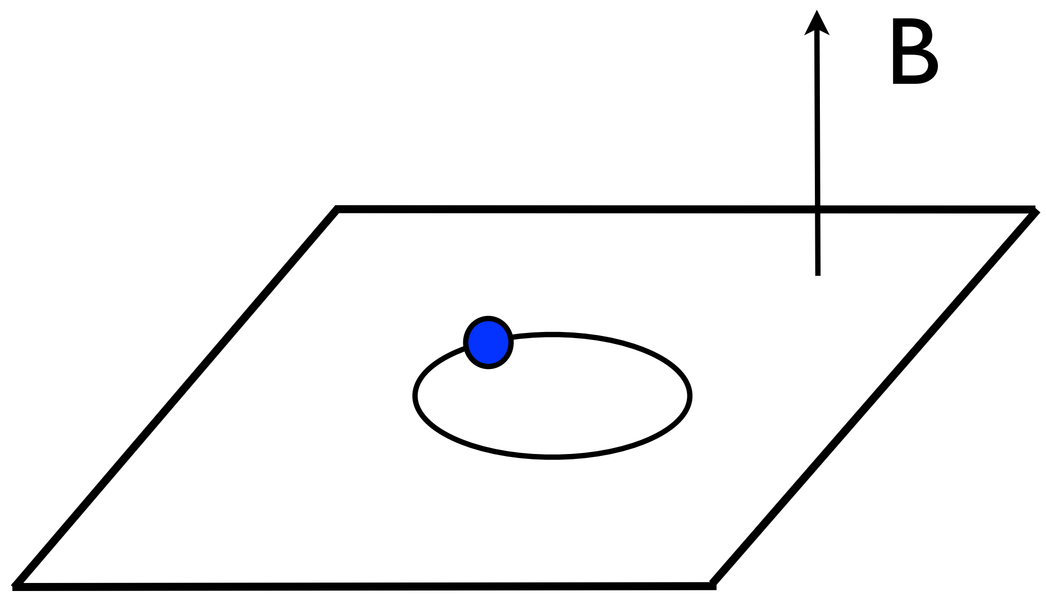
- Consider a massless spin-1/2 particle: 2 chiralities

left right

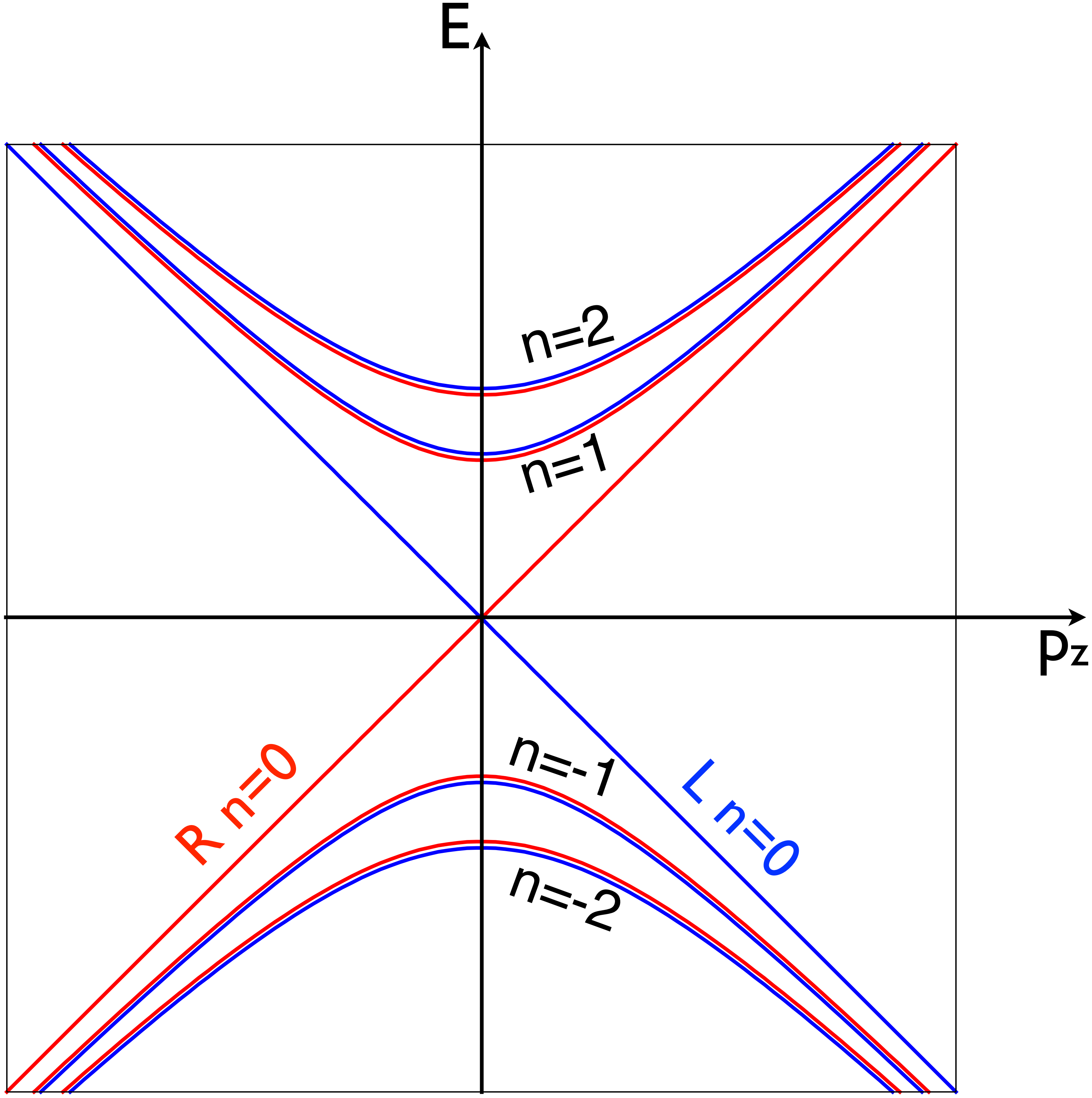
Chirality is preserves in classical theory
But chirality is not conserved in quantum theory: anomalies

Landau levels

- To understand anomalies, we start with quantum mechanics of a massless fermion in a magnetic field



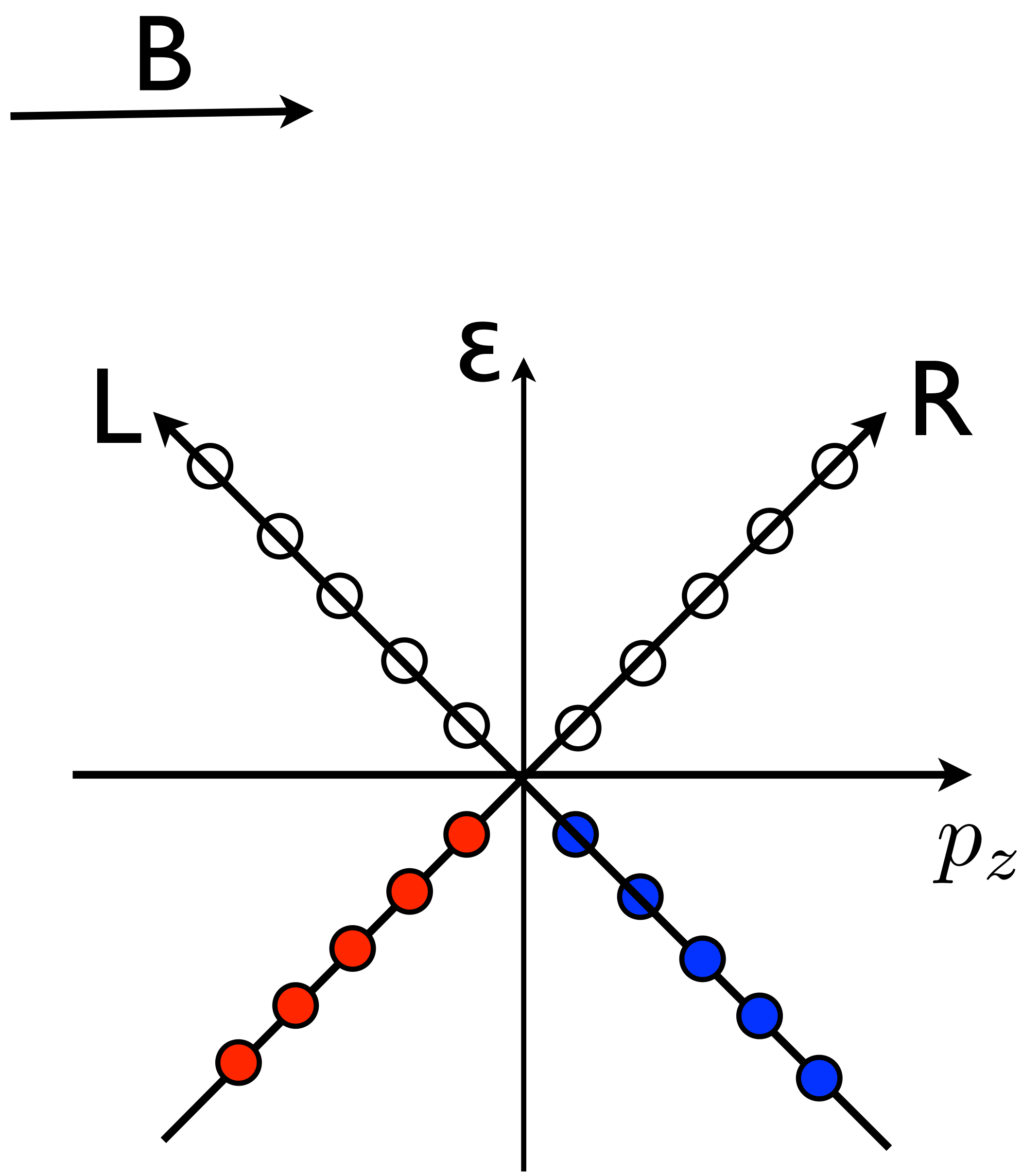
Massless fermion in a magnetic field



$$E^2 = p_z^2 + 2nB$$

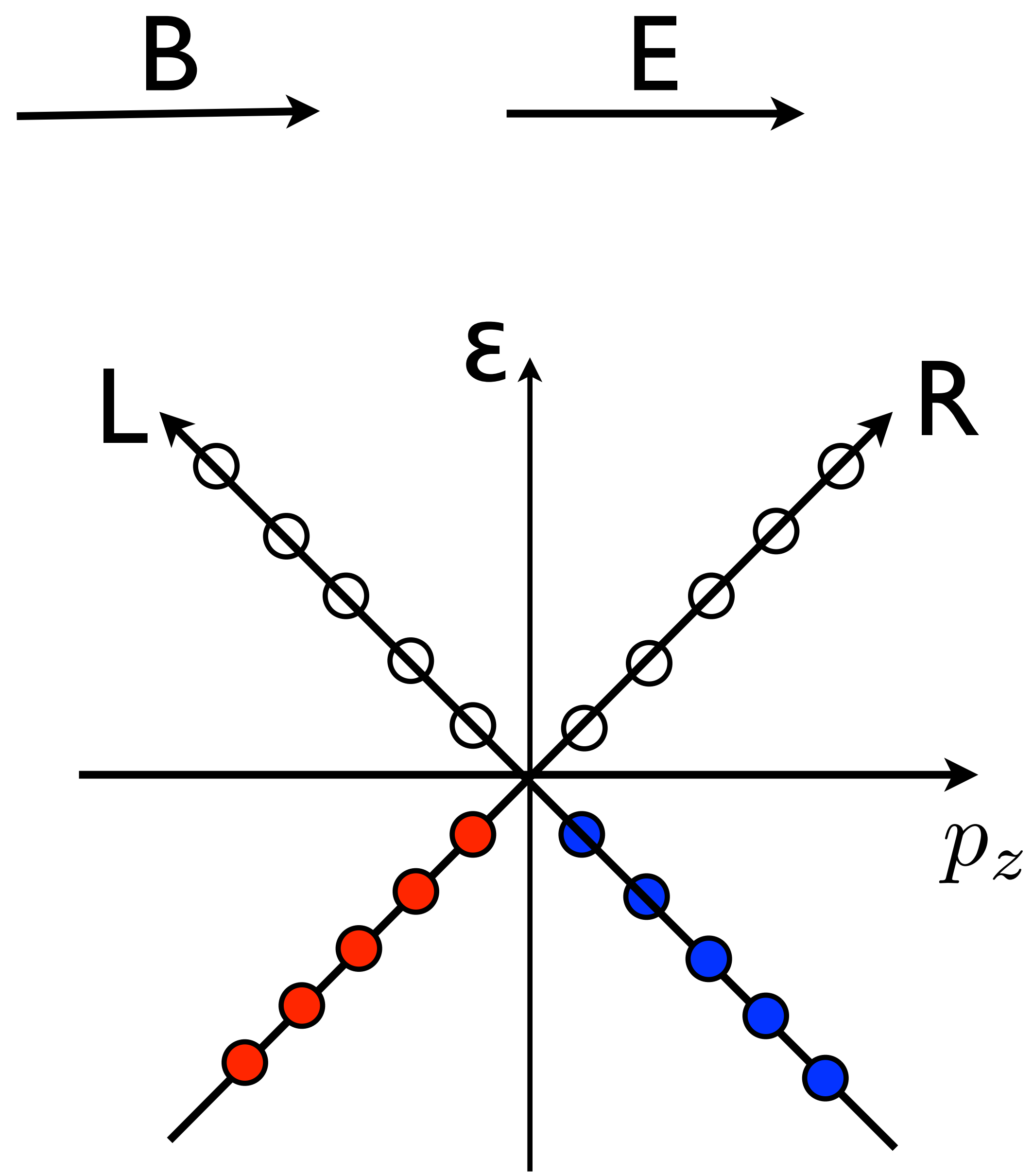
Anomalies

Turn on electric field for some duration of time



Anomalies

Turn on electric field for some duration of time



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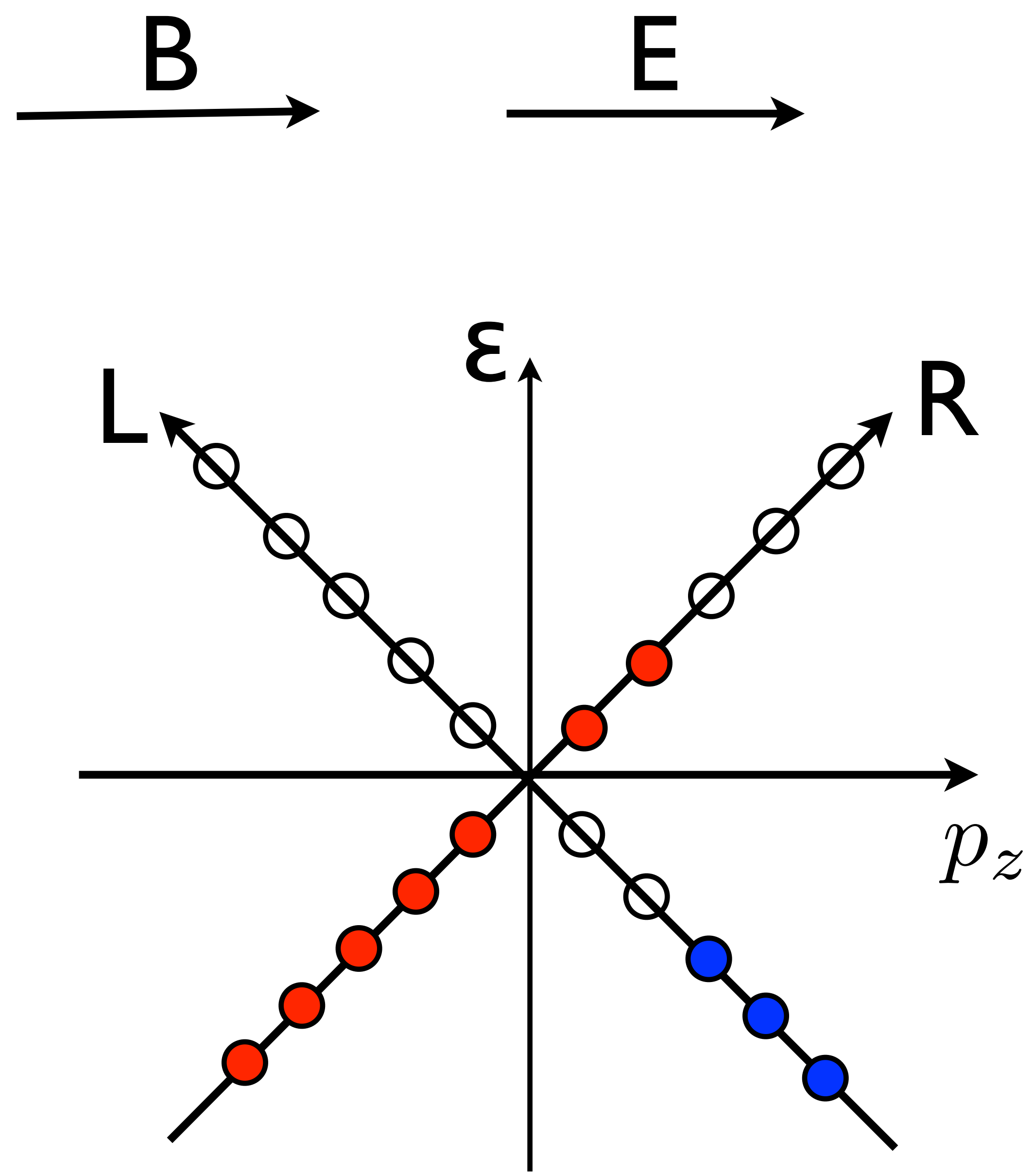

Anomalies

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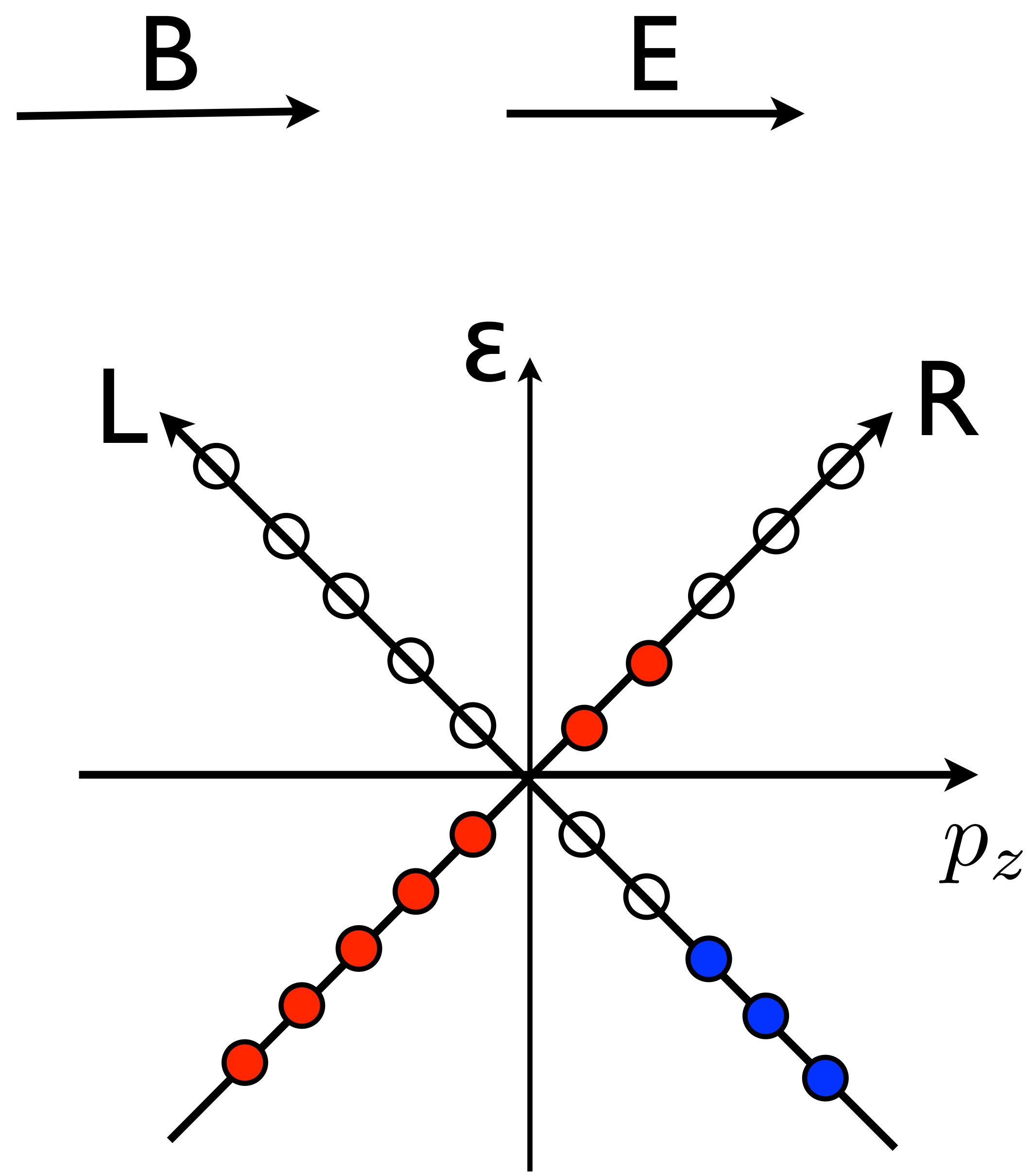
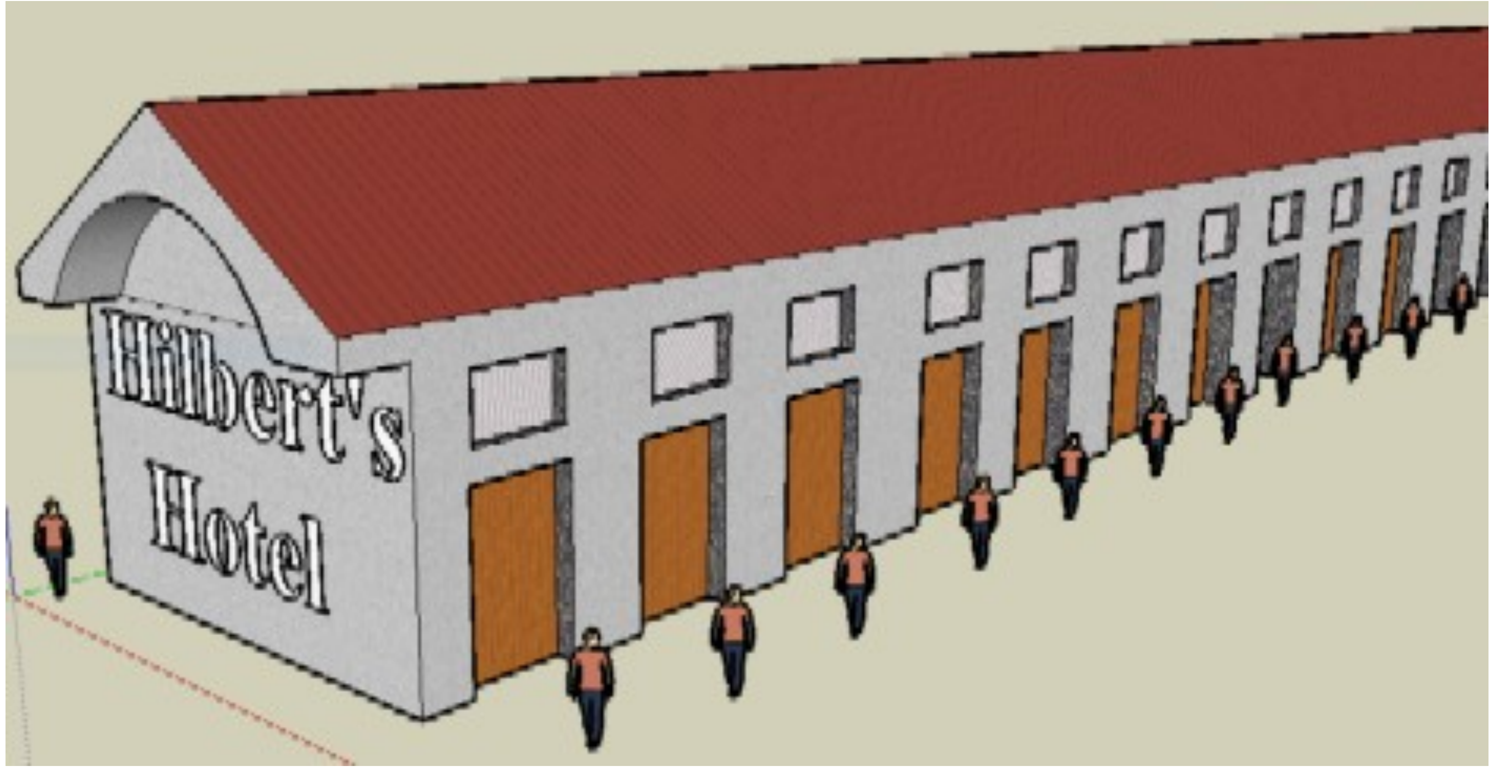
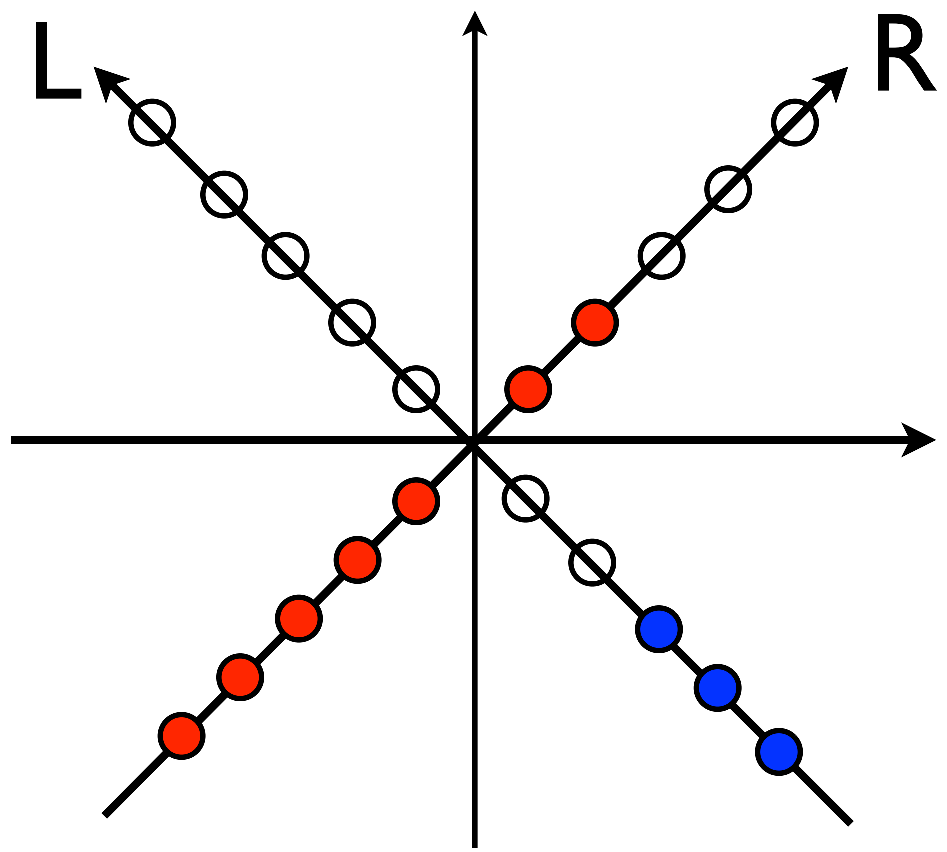


$$\frac{d}{dt}(N_R - N_L) \sim E \cdot B \qquad \partial_\mu j^{5\mu} = \frac{e^2}{4\pi^2}\vec{E}\cdot\vec{B}$$

Hilbert’s hotel



Hilbert's hotel



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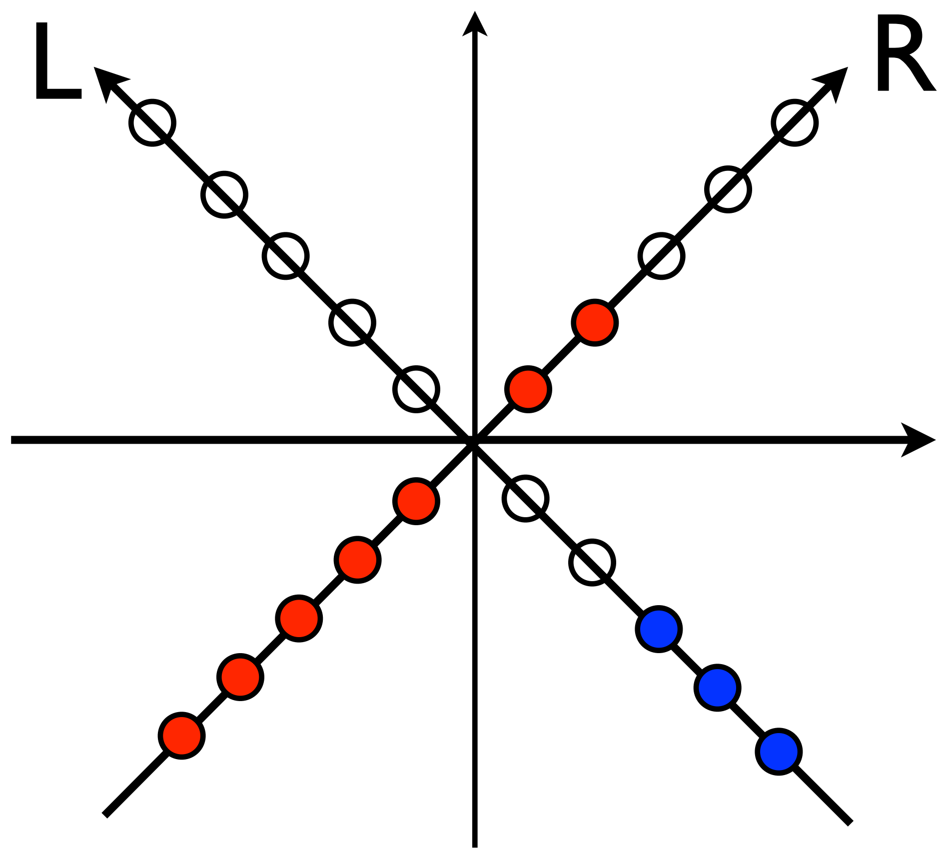
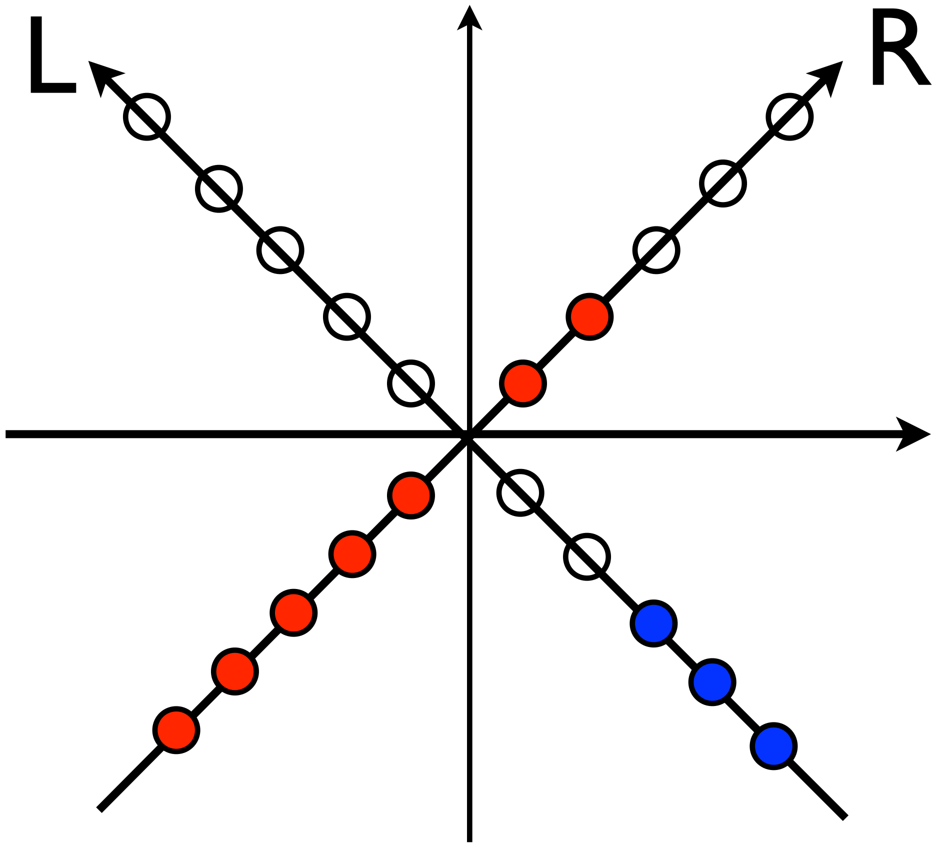


Is $\infty - \infty = 0$ or $\infty - \infty = 4$?

Hilbert's hotel



Is $\infty - \infty = 0$ or $\infty - \infty = 4$?

How to get the right answer is what the understanding of anomaly provides

Anomalies and hydrodynamics

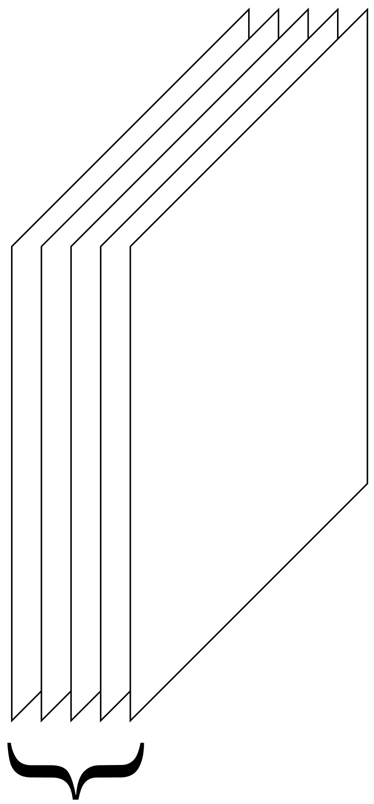
- A full understanding of anomalies in quantum field theory was achieved ~ 1980s
- In general not all global symmetries can be gauged at the same time (coupling to nondynamical background fields is OK)
- In the Standard Model: anomaly cancellation for gauged current, but e.g. baryon number is not exactly conserved

Hydrodynamics with anomalous symmetries? (neutrino gas): difficult using traditional methods

a convenient technique for combining hydro and anomalies: gauge-gravity duality

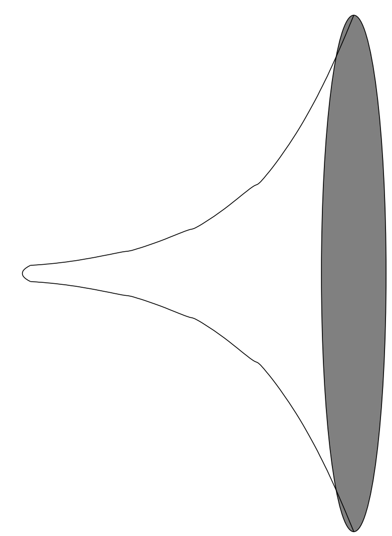
Gauge/gravity duality ("holography")

Maldacena (1997): duality between QFT and string theory



N

$=$



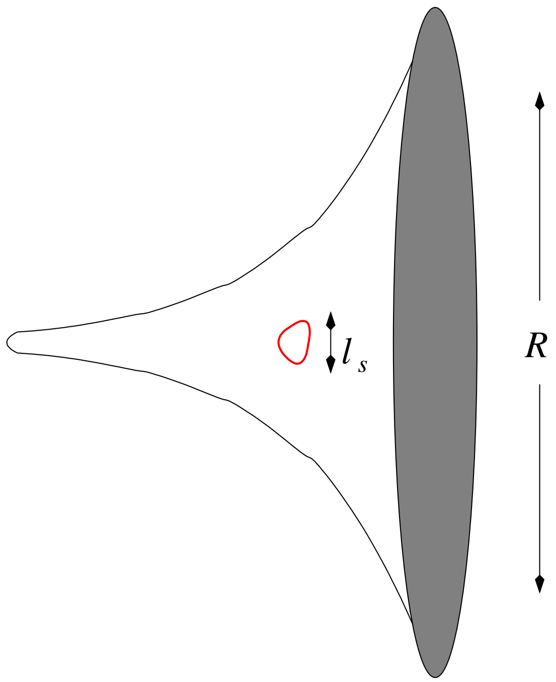
N=4 super Yang-Mills theory

string theory in $AdS_5 x S^5$ space

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + d\vec{x}^2) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2$$

Duality as a tool for QFT

- Gauge/gravity duality is particularly useful in the strong coupling regime of QFT



$$g^2 N_c = \frac{R^4}{\ell_s^4}$$

$g^2N_c >> 1$: string theory becomes gravity

Difficult regime in field theory = easy in string theory

Gauge-gravity duality at finite temperature

- Soon gauge/gravity duality was generalized to finite temperature
- Quark-gluon plasma (in N=4 SYM theory) = black hole in AdS space
- Entropy of the quark gluon plasma = entropy of black hole

Hydrodynamics from BHs

- Around 2001 connection between black hole physics and hydrodynamics was found Kovtun, Policastro, Starinets, Son...
- Dynamics of black hole horizon is described by fluid dynamic equation: fluid-gravity correspondence
- Allow a simple incorporation of anomaly in hydrodynamics

$$S = \frac{1}{8\pi G}\int d^5x\,\sqrt{-g}\left(R - 12 - \frac{1}{4}F_{AB}^2 + \frac{4\kappa}{3}\epsilon^{LABCD}A_L F_{AB}F_{CD}\right)$$

Surprise: two new effects

Erdmenger, Haack, Kaminski, Yarom (2008)

Banerjee, Bhattacharya, Bhattacharyya, Dutta, Loganayagam, Surówka (2008)

$$\vec{j}_5 = \underbrace{\rho_5 \vec{v}}_{\text{advection}} - \underbrace{D\vec{\nabla}\rho_5}_{\text{diffusion}} + \underbrace{\xi_B \vec{B}}_{\text{Chiral magnetic effect (CME)}} + \underbrace{\xi \vec{\nabla} \times v}_{\text{Chiral vortical effect (CVE)}}$$

Chiral separation by rotation?

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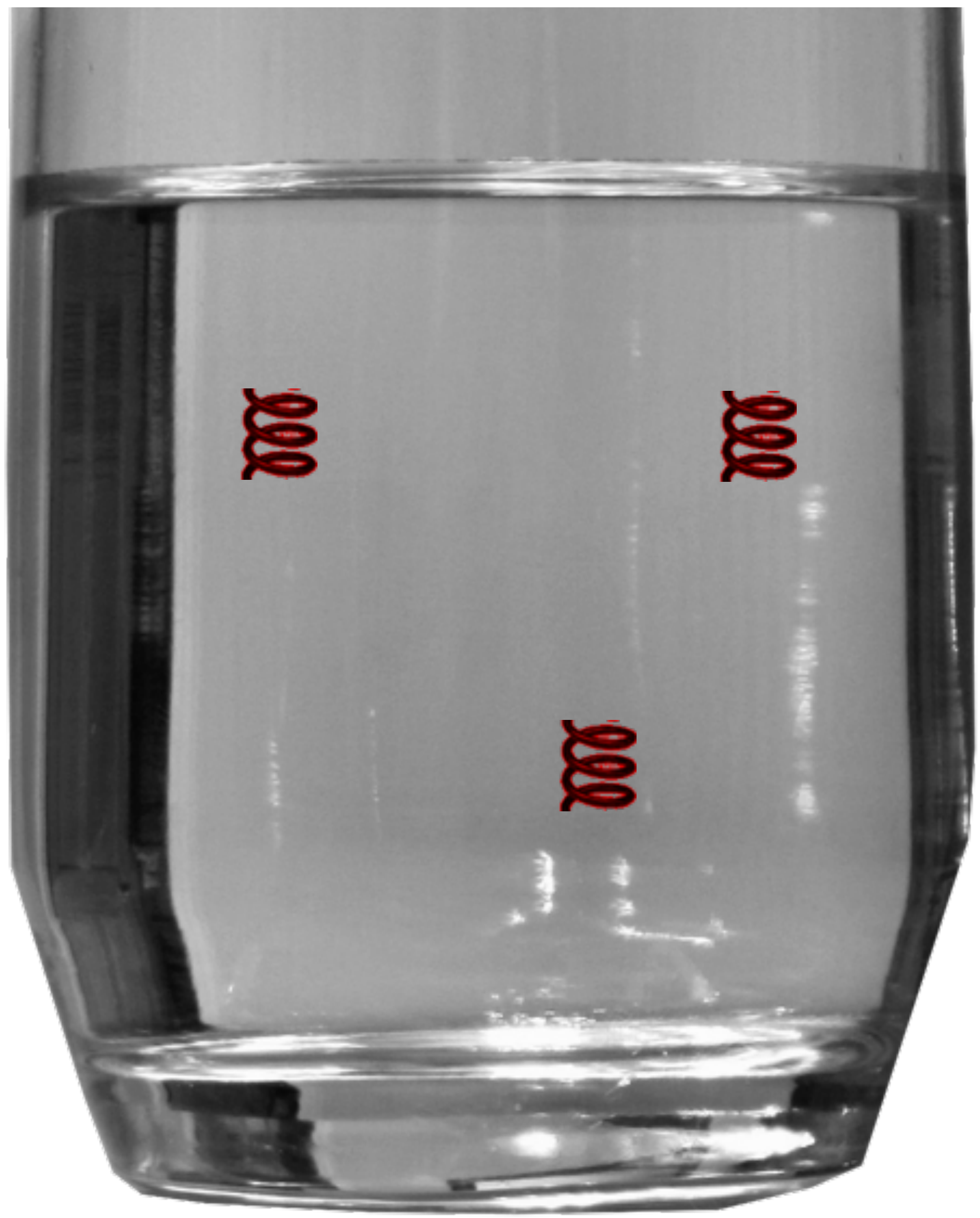

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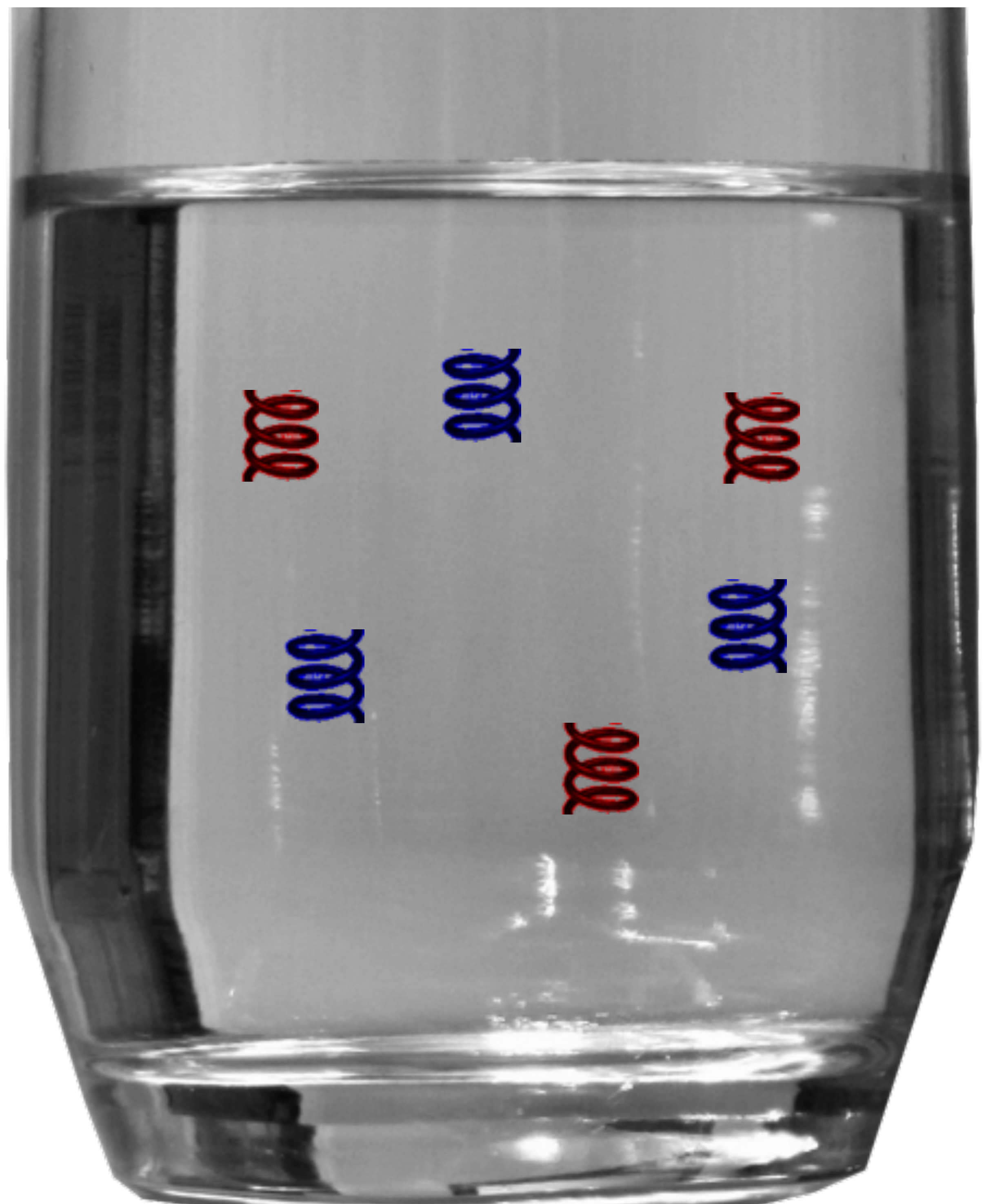
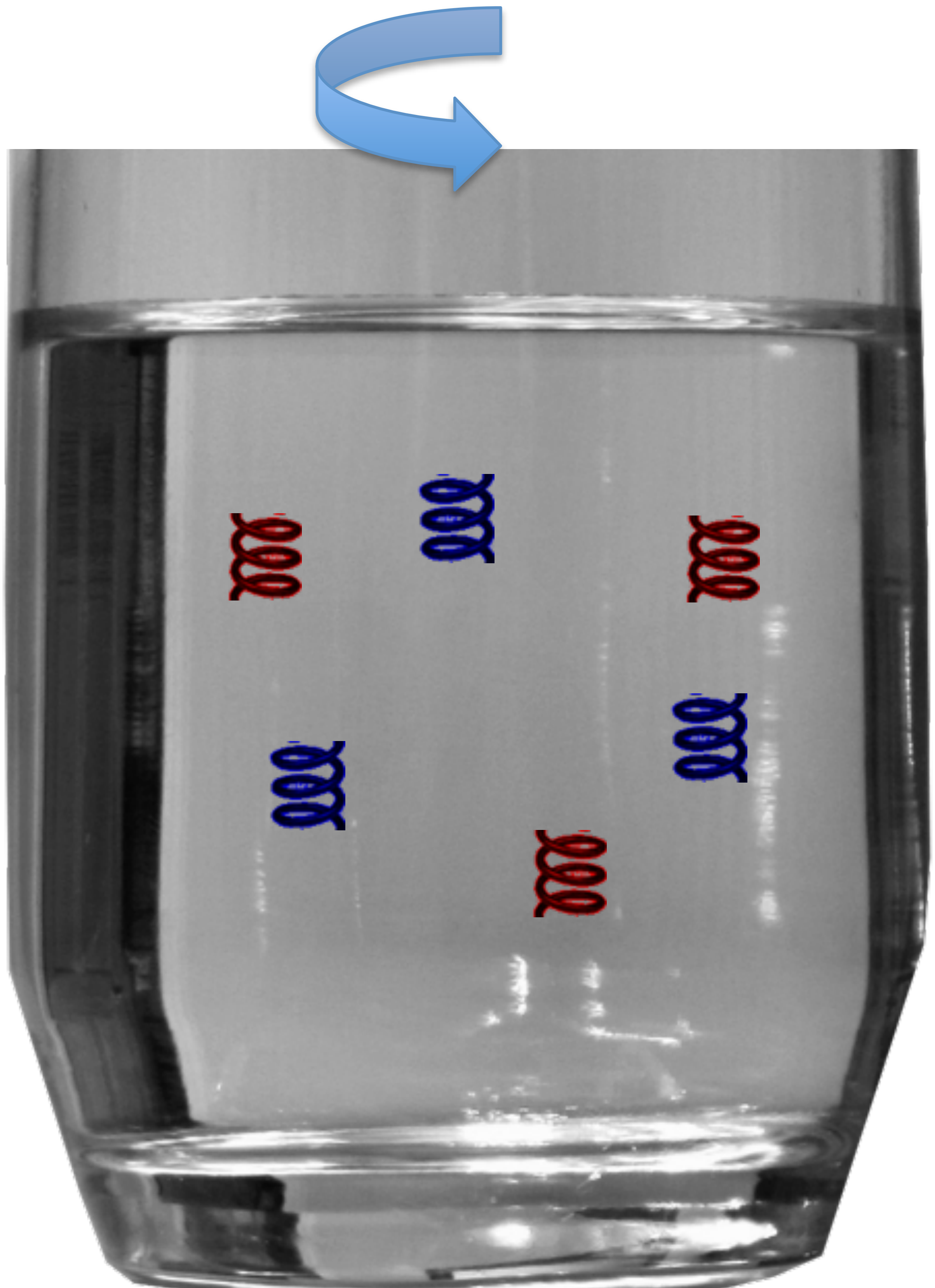
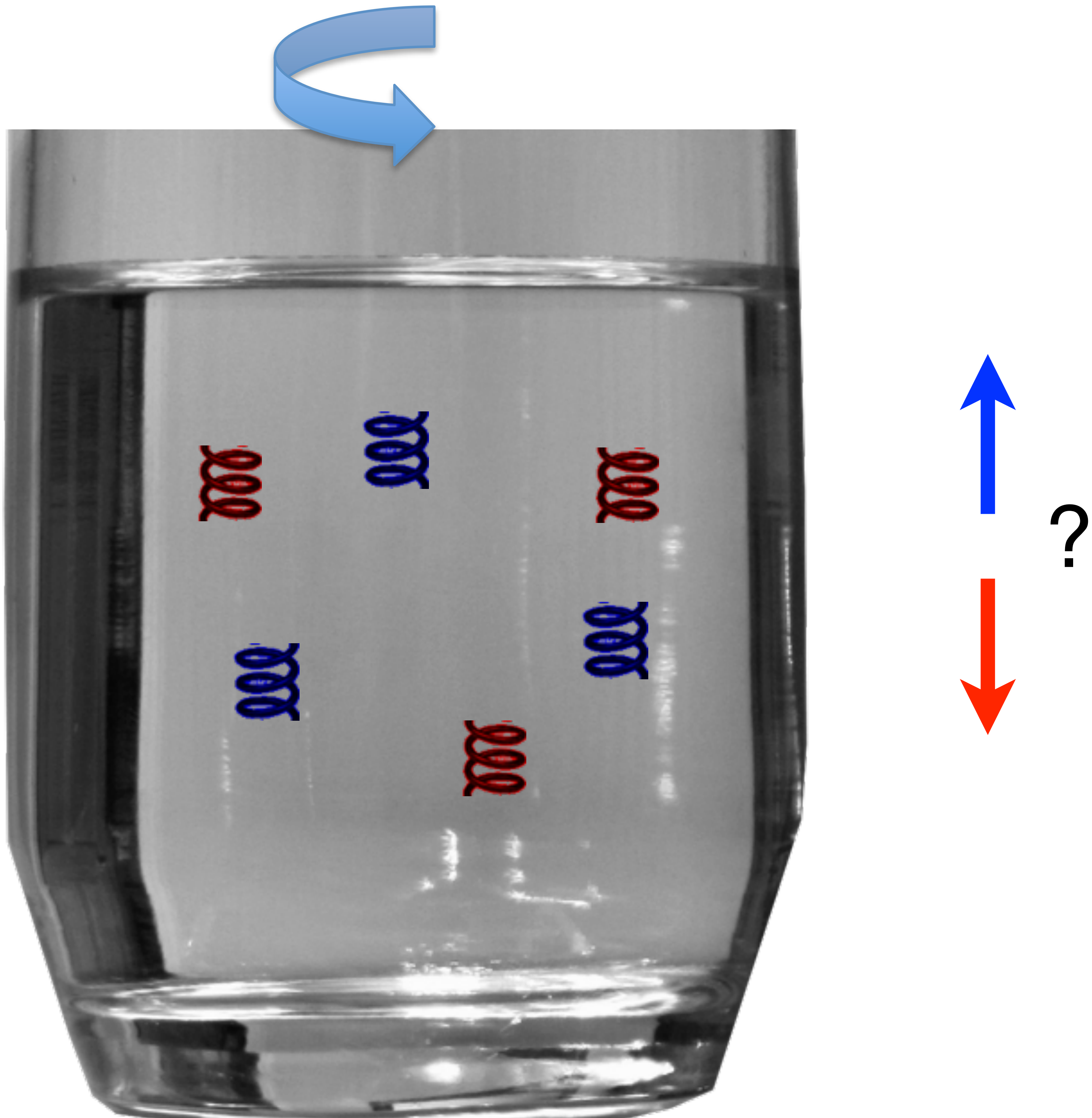


Chiral separation by rotation?



Chiral separation by rotation?



A stronger objection

- There is no mention of a this term in volume 6 of Landau and Lifshitz (Fluid Mechanics) and in any of the 10 volumes !

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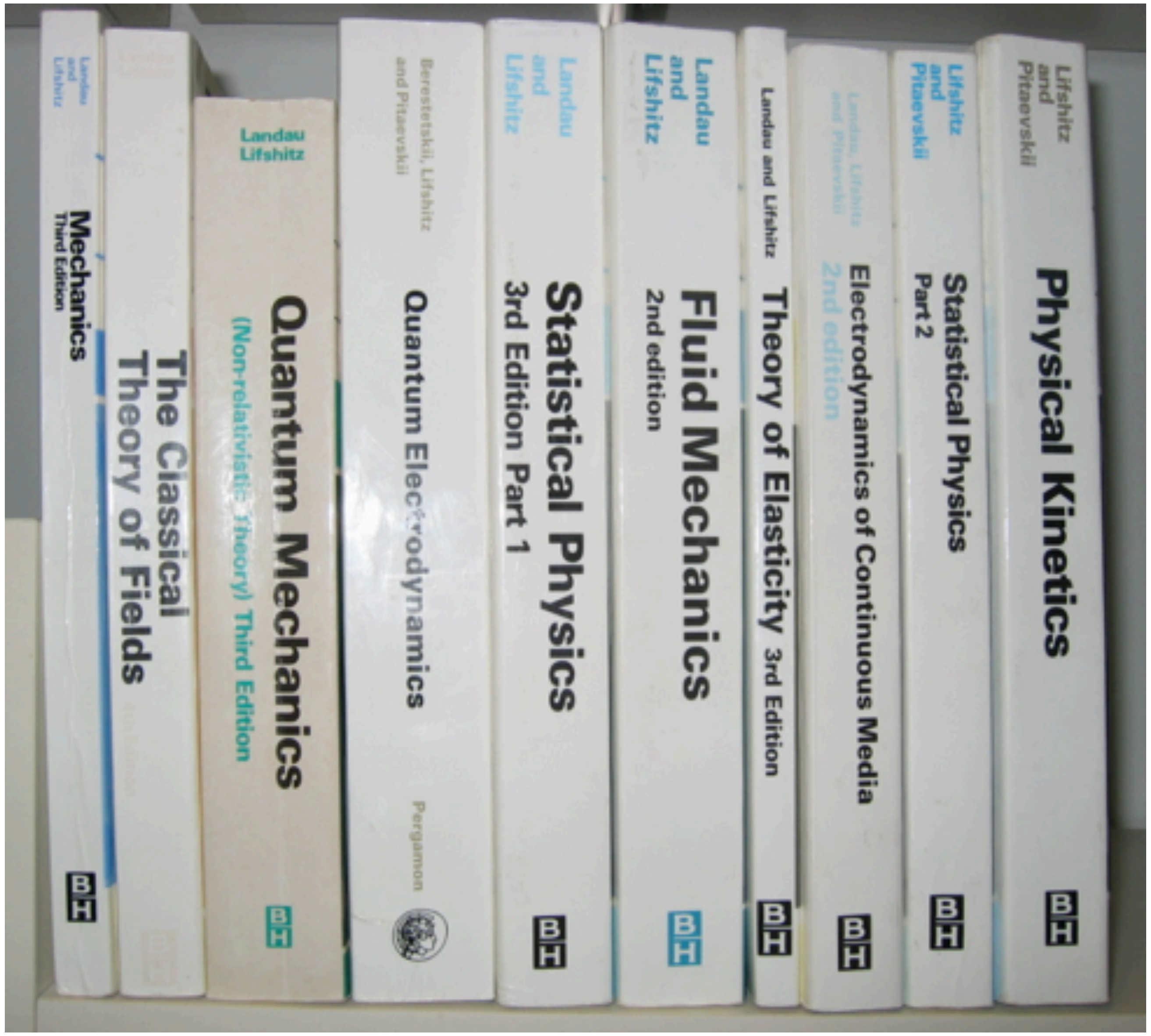


The entropy argument

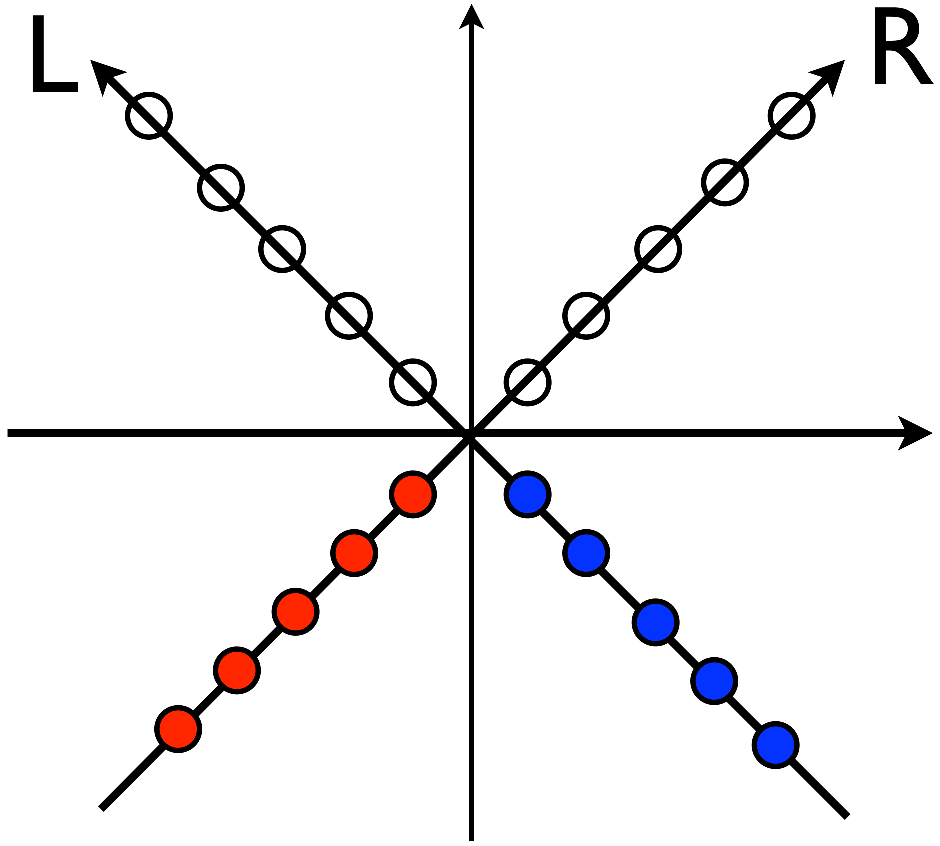
- The hydrodynamic equation is not time-reversal invariant
- There must exist an entropy, expressible in terms of hydrodynamic variables (local velocity, T, etc.) which does not decrease with time
- This condition seems to allows for only shear viscosity, bulk viscosity, thermal conductivity (charge diffusion cofficient) independent of whether spatial parity is broken or not
- Thus sugar in the cup would not separate

Resolution

- But one can check from the positivity of entropy production that
  - vorticity-induced current is not only allowed, but is required if anomaly is present
  - the coefficient governing the chiral vortical effect is related to anomalies Surówka, DTS 2009

$$\xi_{\rm axial} = \underbrace{C}_{\text{anomaly coefficient}}\mu^2 \qquad + \cdots$$

The cousin chiral magnetic effect can be explained in picture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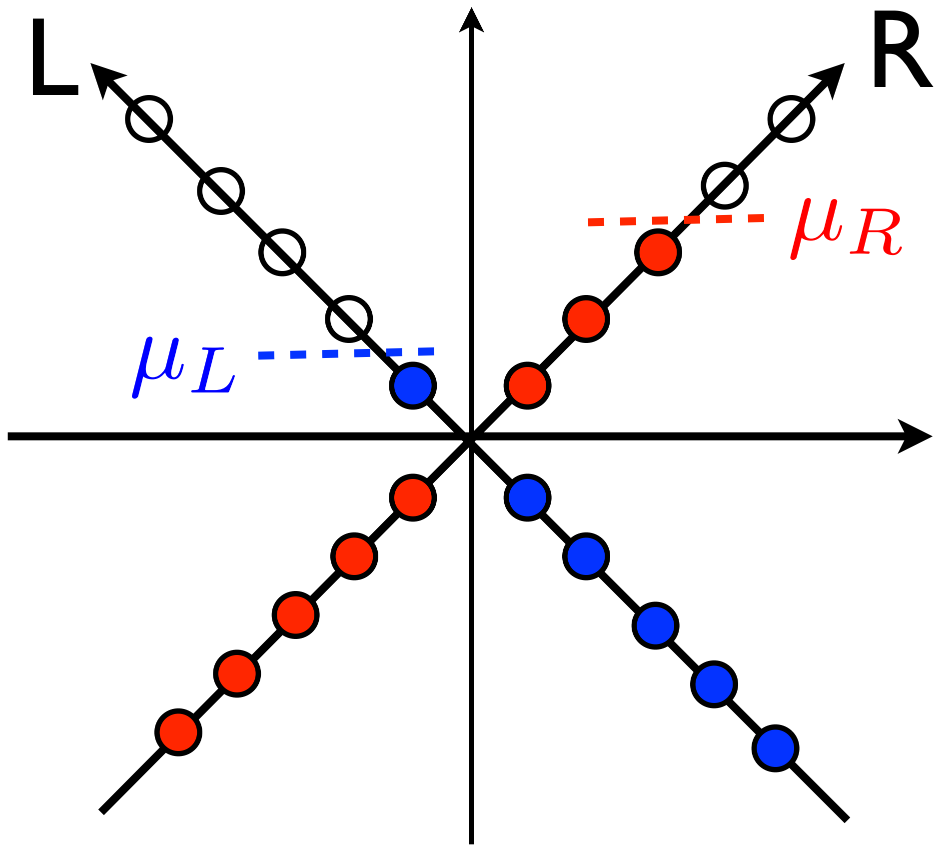
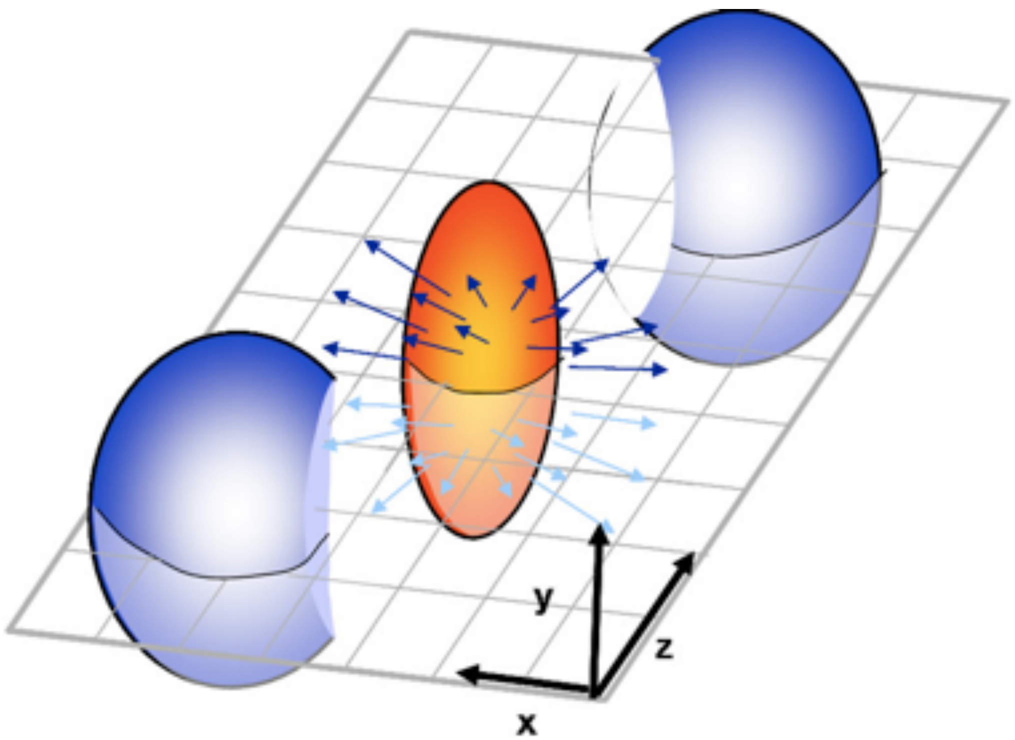
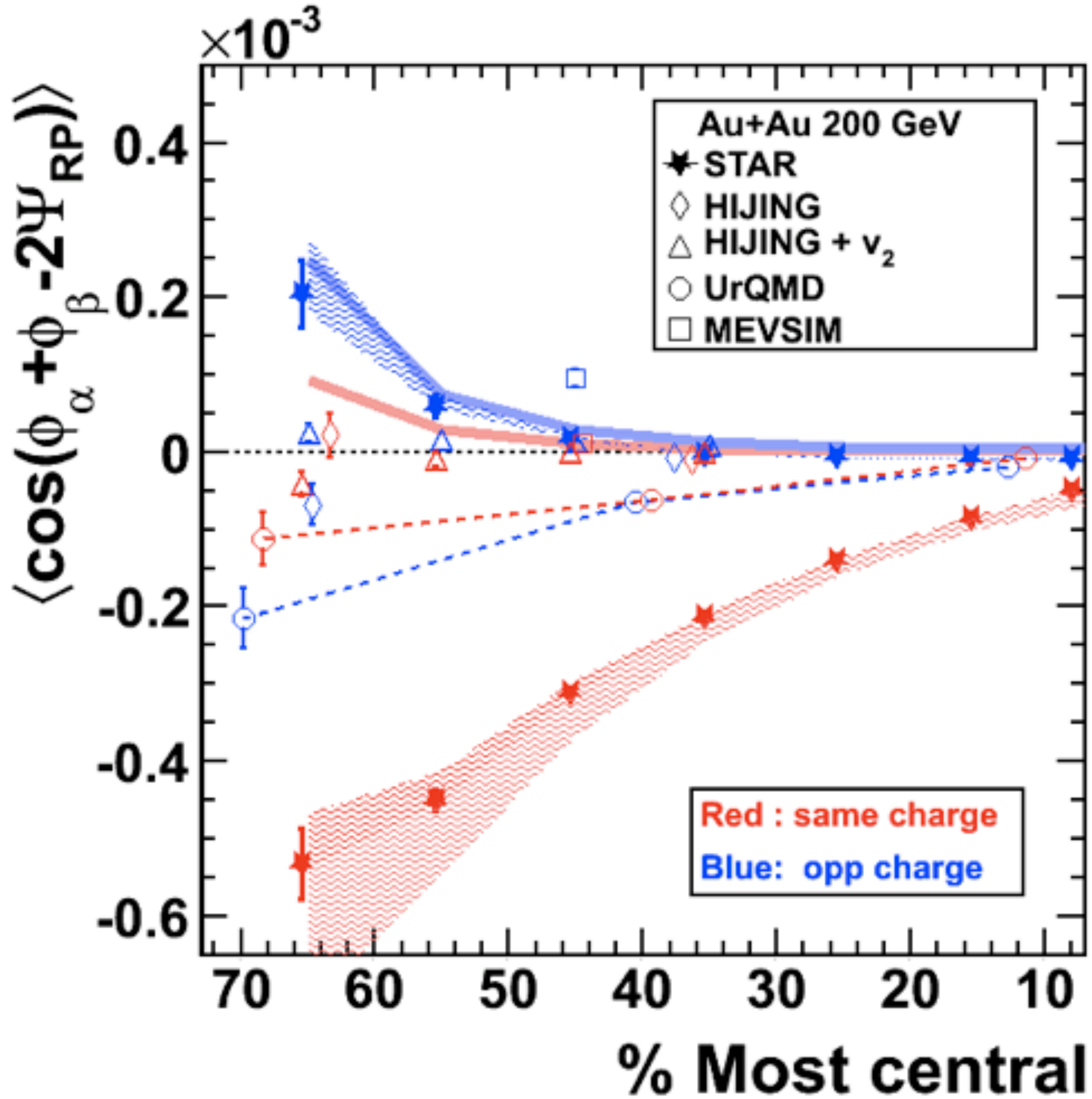


$$\mathbf{j} = \frac{e^2}{4\pi^2}(\mu_R - \mu_L)\mathbf{B}$$

valid also with interactions

Chiral magnetic effect at RHIC?





Abelev et al. PRL 2009 (arxiv:0909.1739)

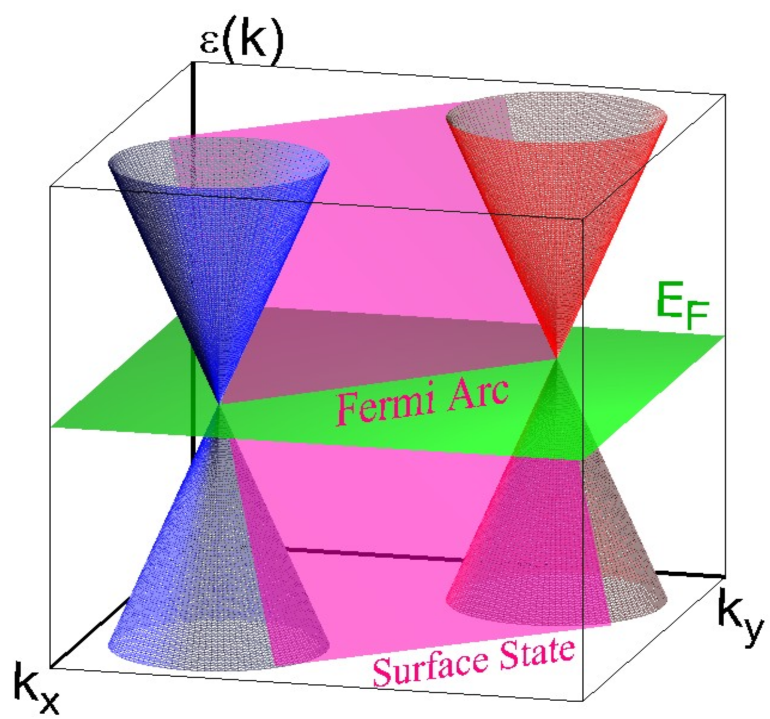
π^+s like to travel in the same directions, π^+ and π^- in opposite directions

Chiral magnetic effect in heavy ion collsions?

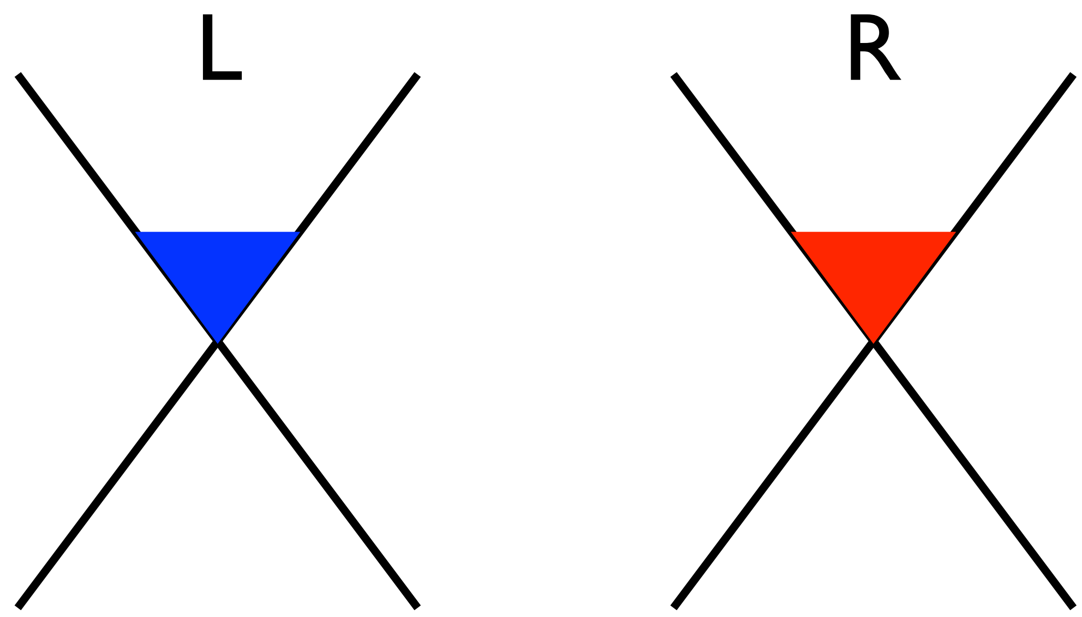
- If the quark gluon plasma was created with an asymmetry in the number of left- and right-handed quarks (either sign)
- then the magnetic field leads to an electric current and may lead to observed effect
- (However, alternative explanations exist)

Dirac or Weyl semimetals

- Solid-state materials with energy spectrum of massless Dirac fermions
- Effects of anomalies may be visible in transport



(Turner, Vishwanath)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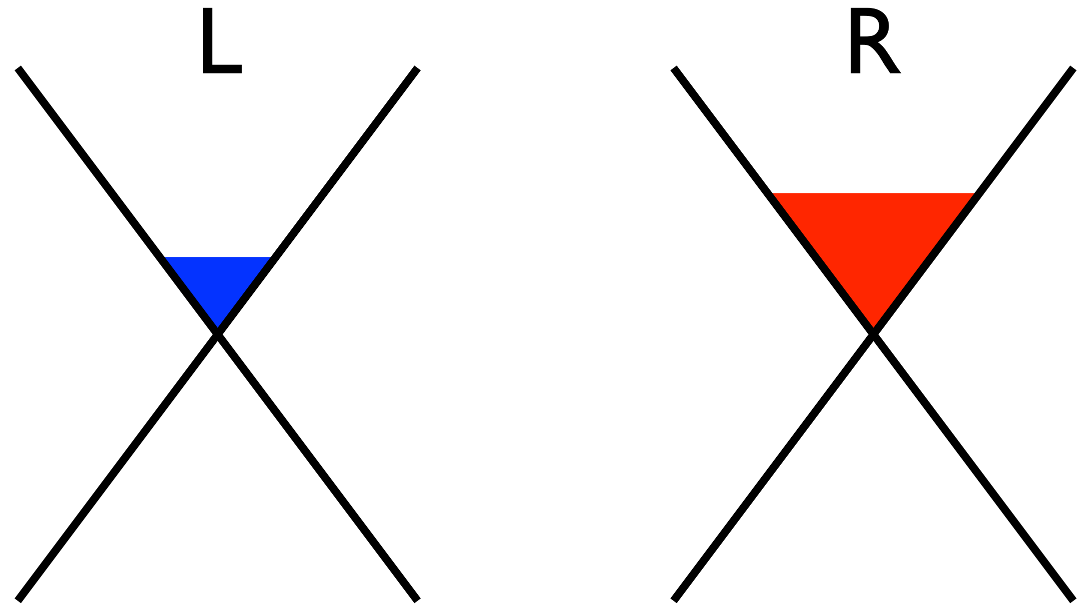


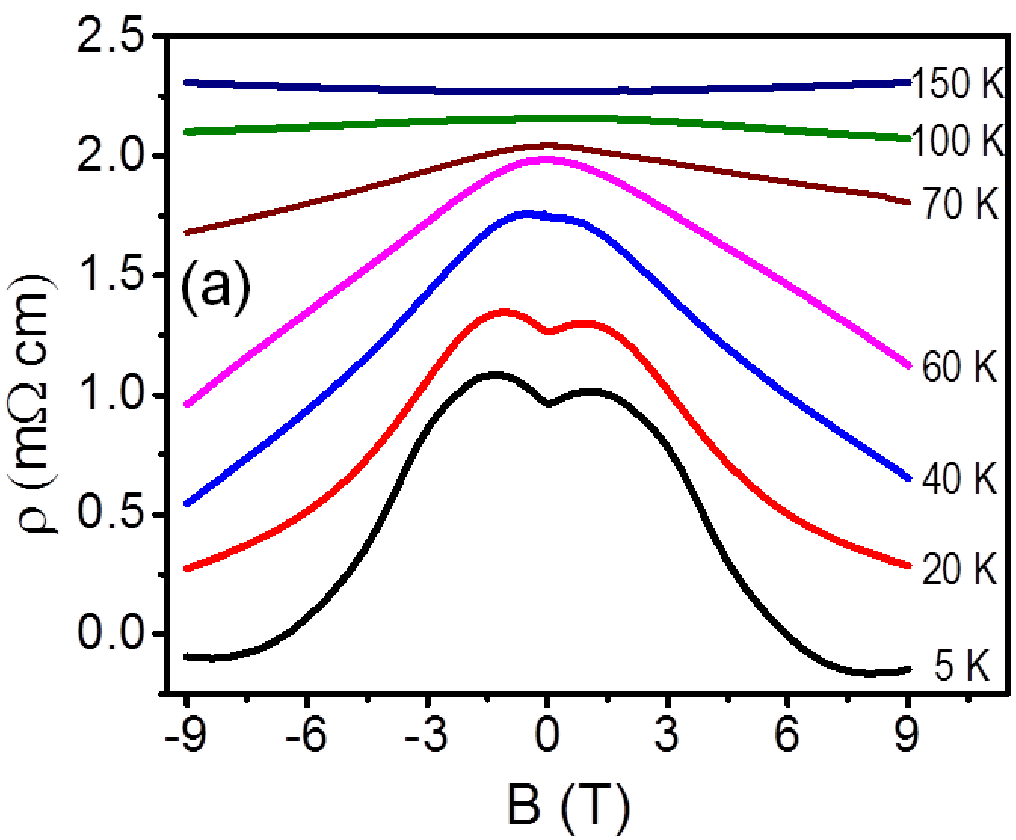
$$\frac{\partial}{\partial t}(n_R - n_L) = E \cdot B + \frac{n_R - n_L}{\tau}$$

$$j \sim (\mu_R - \mu_L)B \sim \tau B^2 E$$

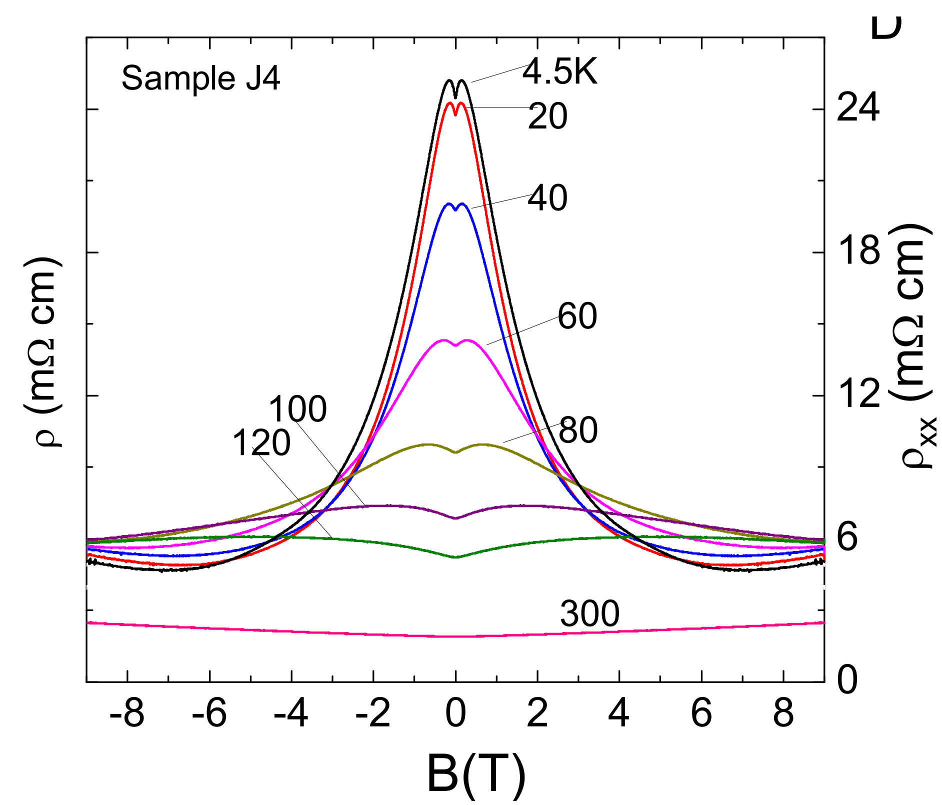
enhancement of current
negative magnetoresistance

(DTS, Spivak 2012; Burkov 2014)

Experimental observation of anomaly in solids



1412.6543 ($ZrTe_5$)



1503.08179 (Na_3Bi)

Unexplained: strong dependence on the angle between electric and magnetic fields

Outlook and conclusions

- Interesting mathematical structures of hydrodynamic theories with anomalies
- Boltzmann equation with anomaly: role of the Berry phase Chen, DTS, Stephanov, Yee, Yin
- First found in string theory, applications in heavy-ion and condensed matter physics
- Fruitful cross-fertilization between subfields of physics