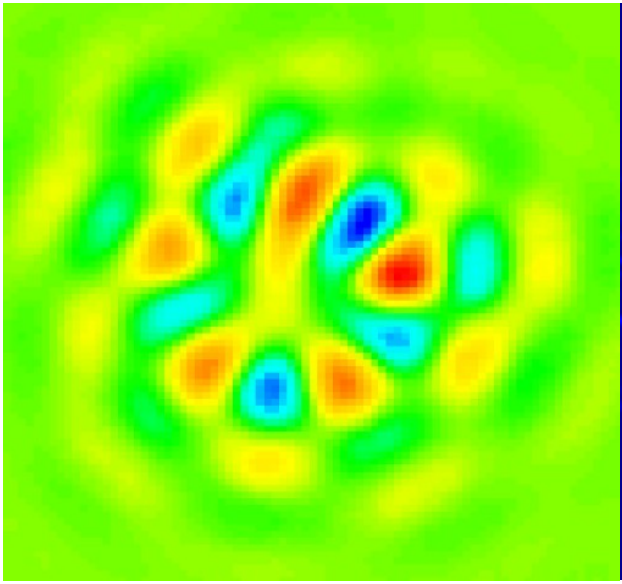
Predicting collective dynamics and instabilities in high-brightness storage ring light sources



Ryan Lindberg

Physicist, Accelerator Operations and Physics Group
Accelerator Systems Division, Argonne National Laboratory

Enrico Fermi Institute Seminar
Presidents’ Day, 2022

Acknowledgements

- Members of the APS-U physics team
  - Tim Berenc
  - Michael Borland
  - John Byrd
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  - Vadim Sajaev
  - Yipeng Sun
  - Uli Wienands
  - Aimin Xiao
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  - Gabriele Bassi (NSLS-II)
  - Alexei Blednykh (NSLS-II)
  - Marco Venturini (ALS/ALS-U)
  - ESRF-EBS physics team
- Computing resources at ANL’s Blues and Bepop clusters, and ASD’s weed cluster
- Funding from the Department of Energy’s Office of Basic Energy Sciences

Outline

- Motivation: bright X-rays for science
- Storage rings for bright X-rays
- How Coulomb collisions impacts design choices
- How design choices impact other collective effects
  - Single bunch dynamics and instabilities
  - Coupled-bunch instabilities
- Conclusion

Motivation: X-rays for science

- X-rays have played an important role in scientific discovery since their discovery
- X-rays are now used to probe many systems:
  - Electronic and magnetic materials
  - Chemical science
  - Life science and medicine
  - Biology and biochemistry
  - Geological and planetary science
  - Nanomaterials
  - ...

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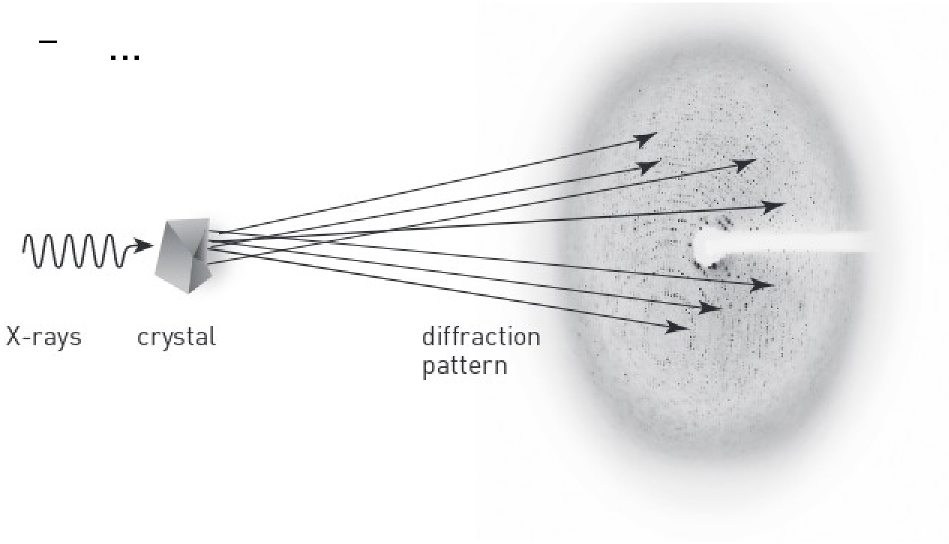

V. Ramakrishnan, T.A. Steitz,and A.E. Yonath
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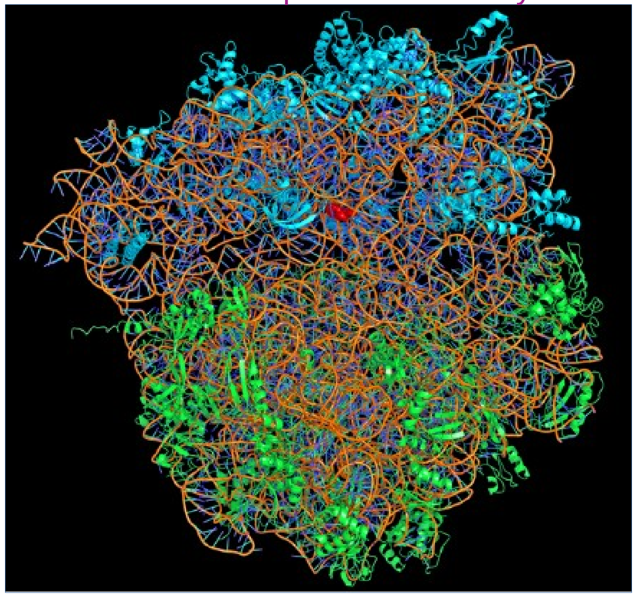

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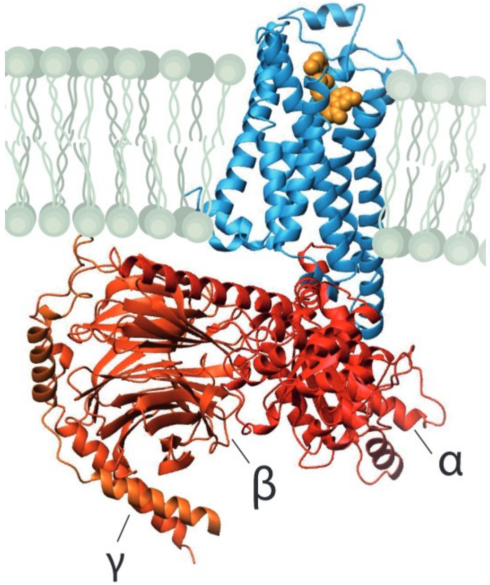

Press release. NobelPrize.org. Nobel Media AB 2019. Fri. 30 Aug 2019.

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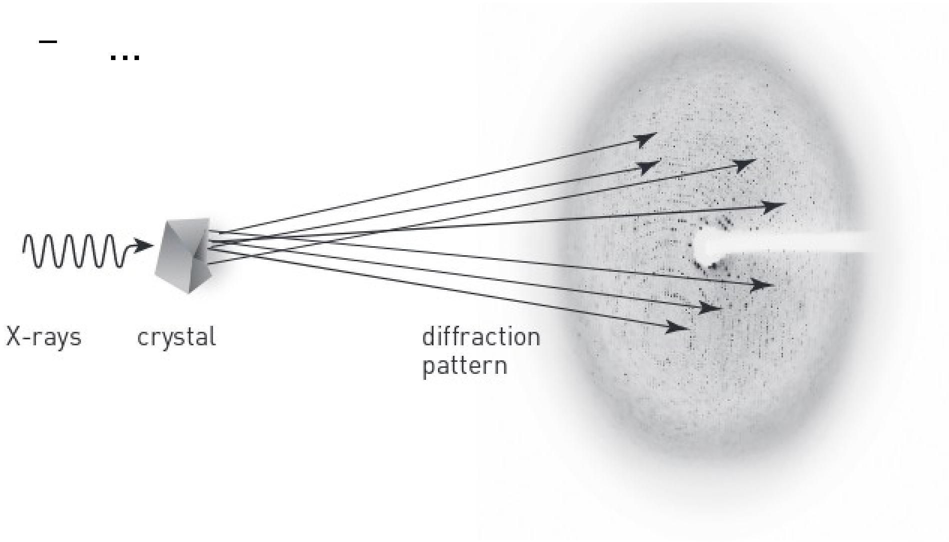
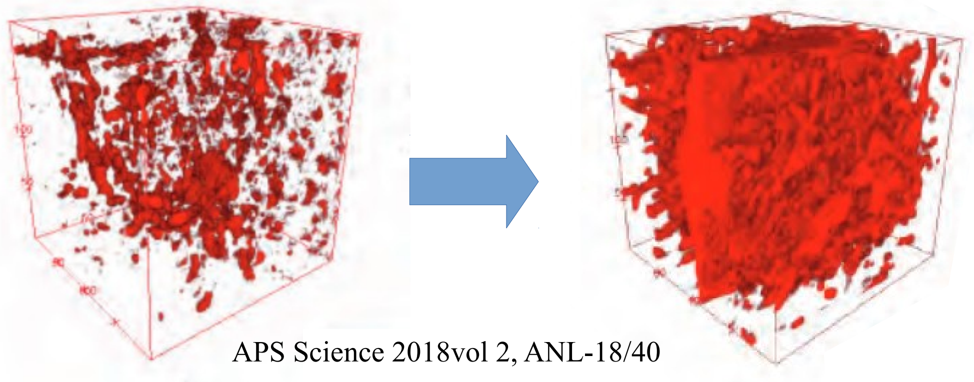


F. Shen et al., ACS Energy Lett. 3, 1056 (2018). ©2018 American Chemical Society
Microstructure-driven failure in Lithium ion batteries



APS Science 2018vol 2, ANL-18/40

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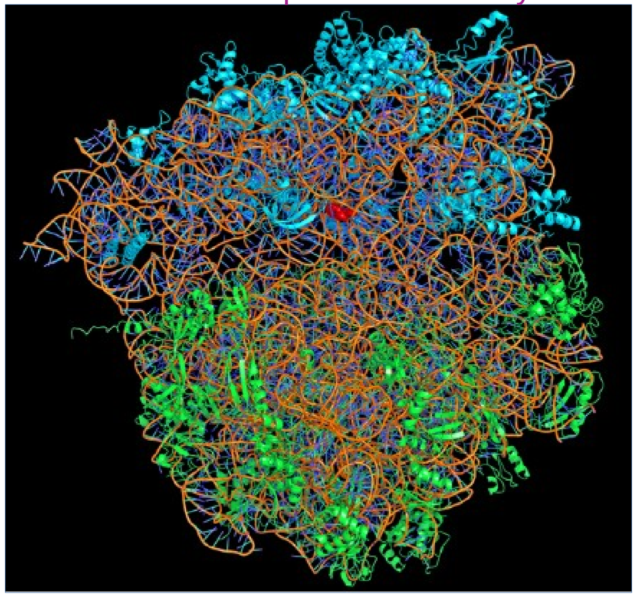

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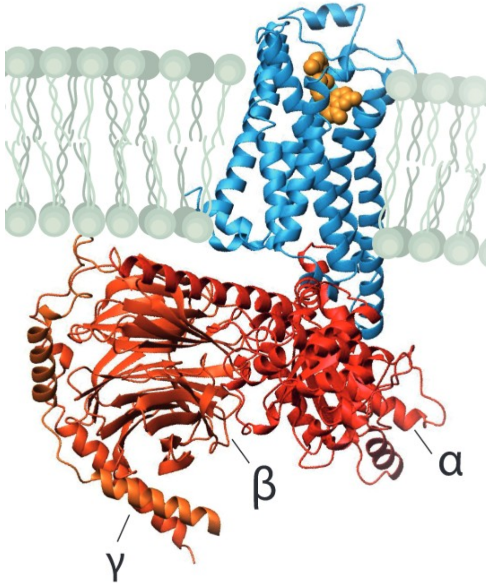

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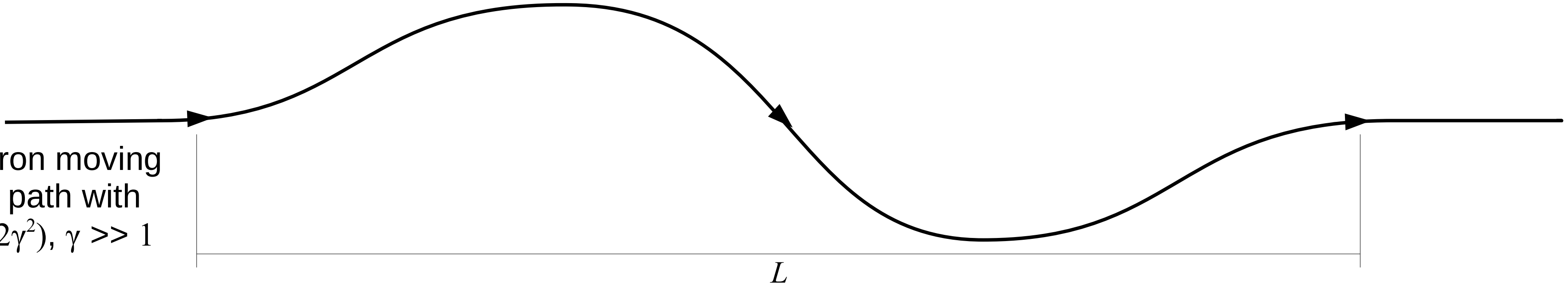
Synchrotron radiation is an intense source of X-rays



Relativistic electron moving along a curved path with speed $v \approx c(1-1/2\gamma^2)$, $\gamma >> 1$

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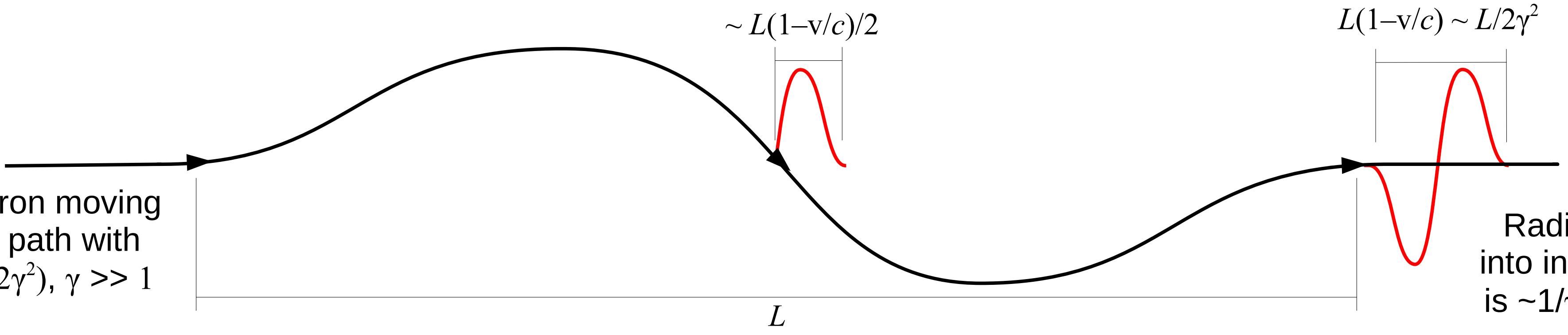

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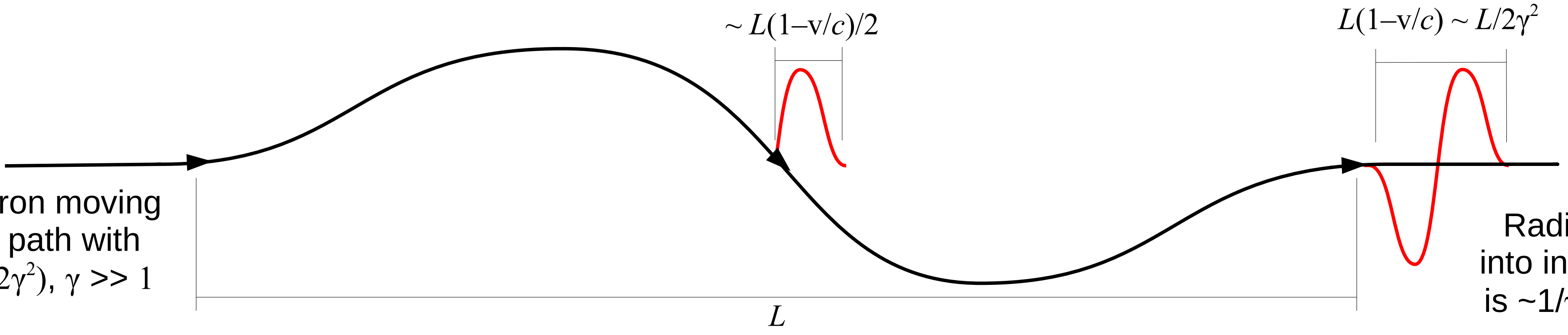

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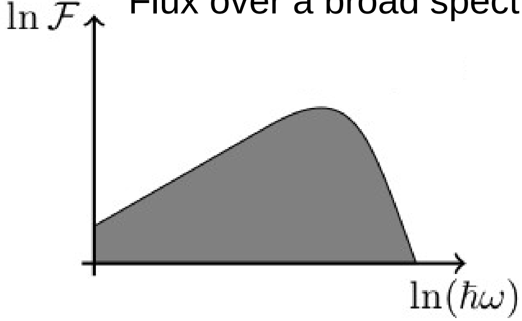
1. Bending magnets

Flux over a broad spectrum



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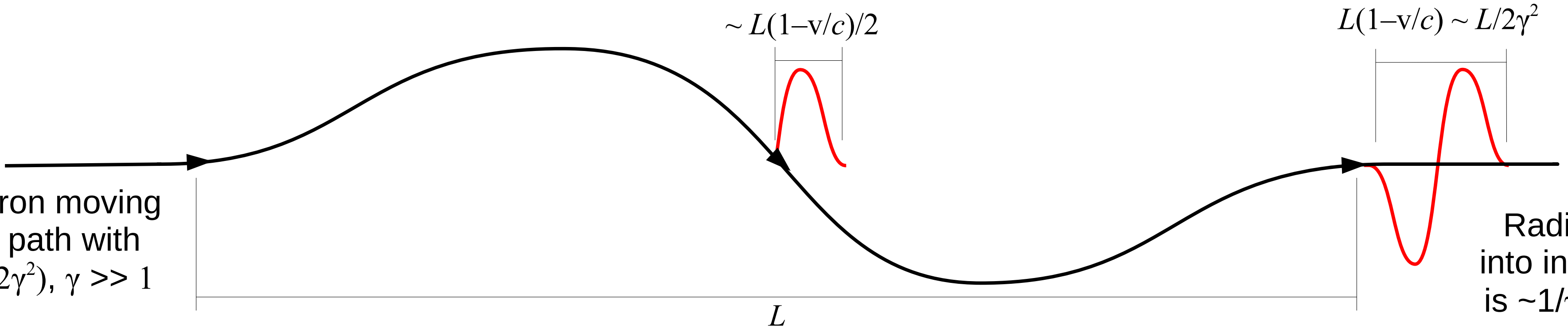

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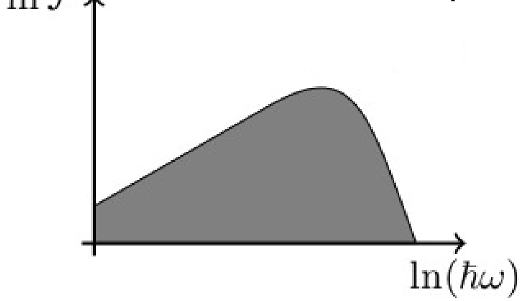
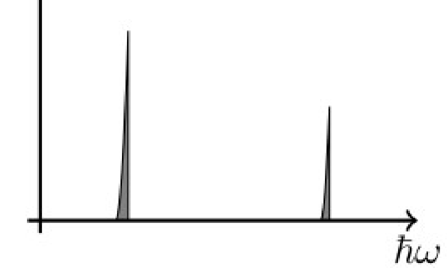
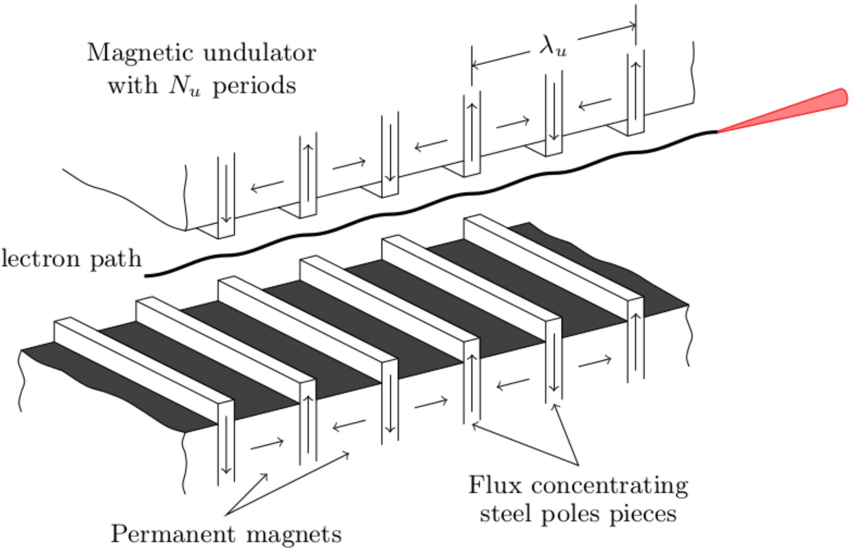


2. Undulators

Flux at harmonics of $\lambda \sim \frac{\lambda_u}{2\gamma^2}$





Light sources are located all over the world

FROM: APS Science 2014, ANL-15/03

X-ray brightness and electron beam emittance

$$\text{X-ray brightness} = \frac{\text{Number of photons}}{\text{6D phase space volume}} = \frac{\text{photons/time}}{(\text{2D area})_x(\text{2D area})_y(\text{Spectral bandwidth})}$$

High brightness → Ability to focus large numbers of photons to a small spot
→ Large photon flux through an aperture
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Proportional to total electron beam current

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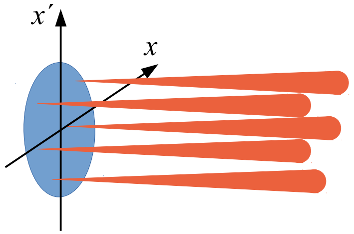
2D area is given by a convolution of the phase space areas associated with the X-rays and electrons



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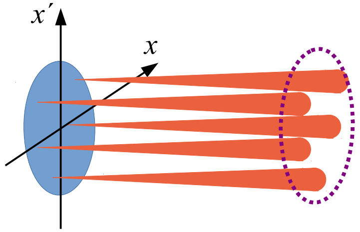

Phase space area of radiation

$$(\text{2D Area})_x \approx 2\pi\left(\boxed{\frac{\lambda}{4\pi}} + \boxed{\varepsilon_x}\right)$$

Phase space area of the electron beam = emittance

ε_x = (RMS spatial size)(RMS angular spread)

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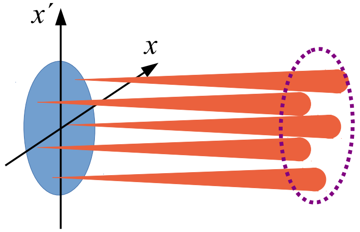

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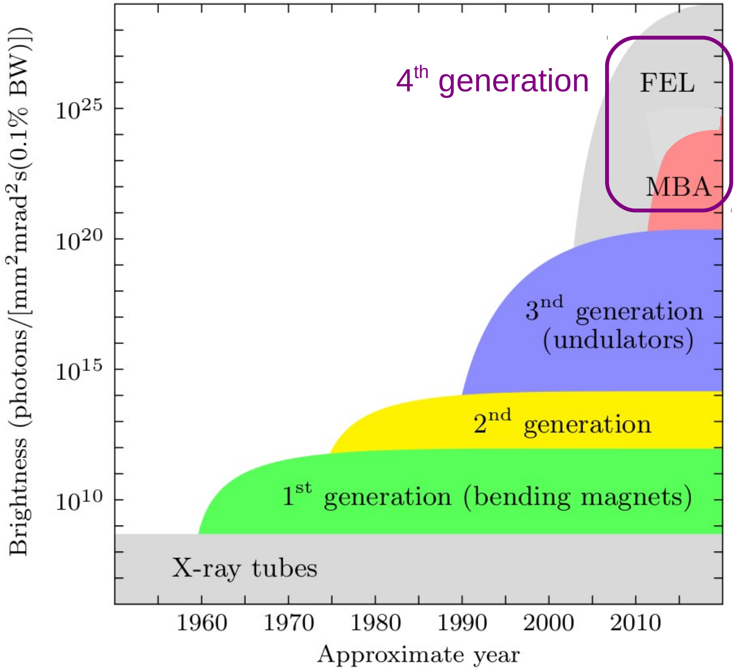
Maximizing X-ray brightness means minimizing electron beam emittance!



Ryan Lindberg -- Collective dynamics and instabilities in high-brightness storage rings -- 2/21/2022

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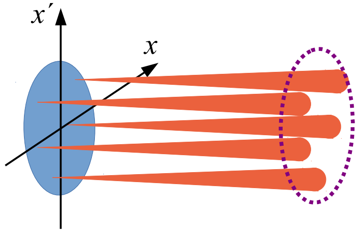

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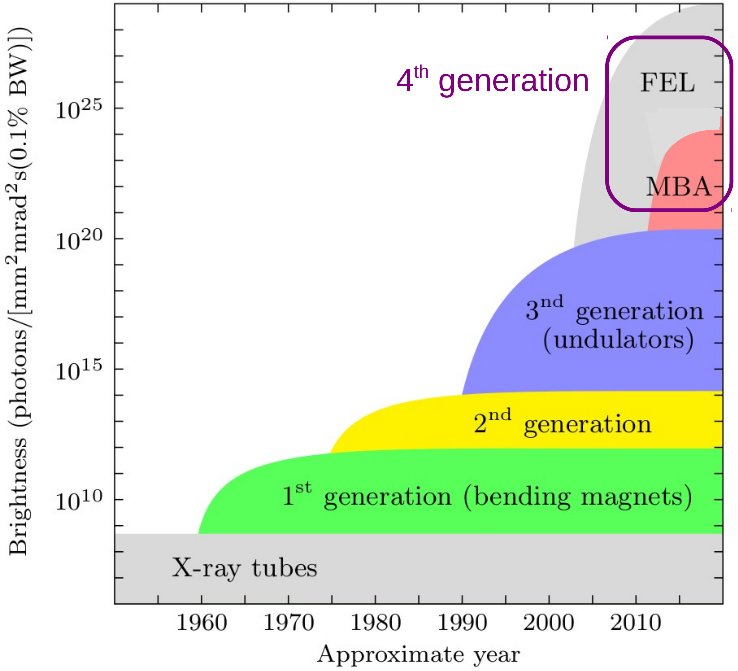
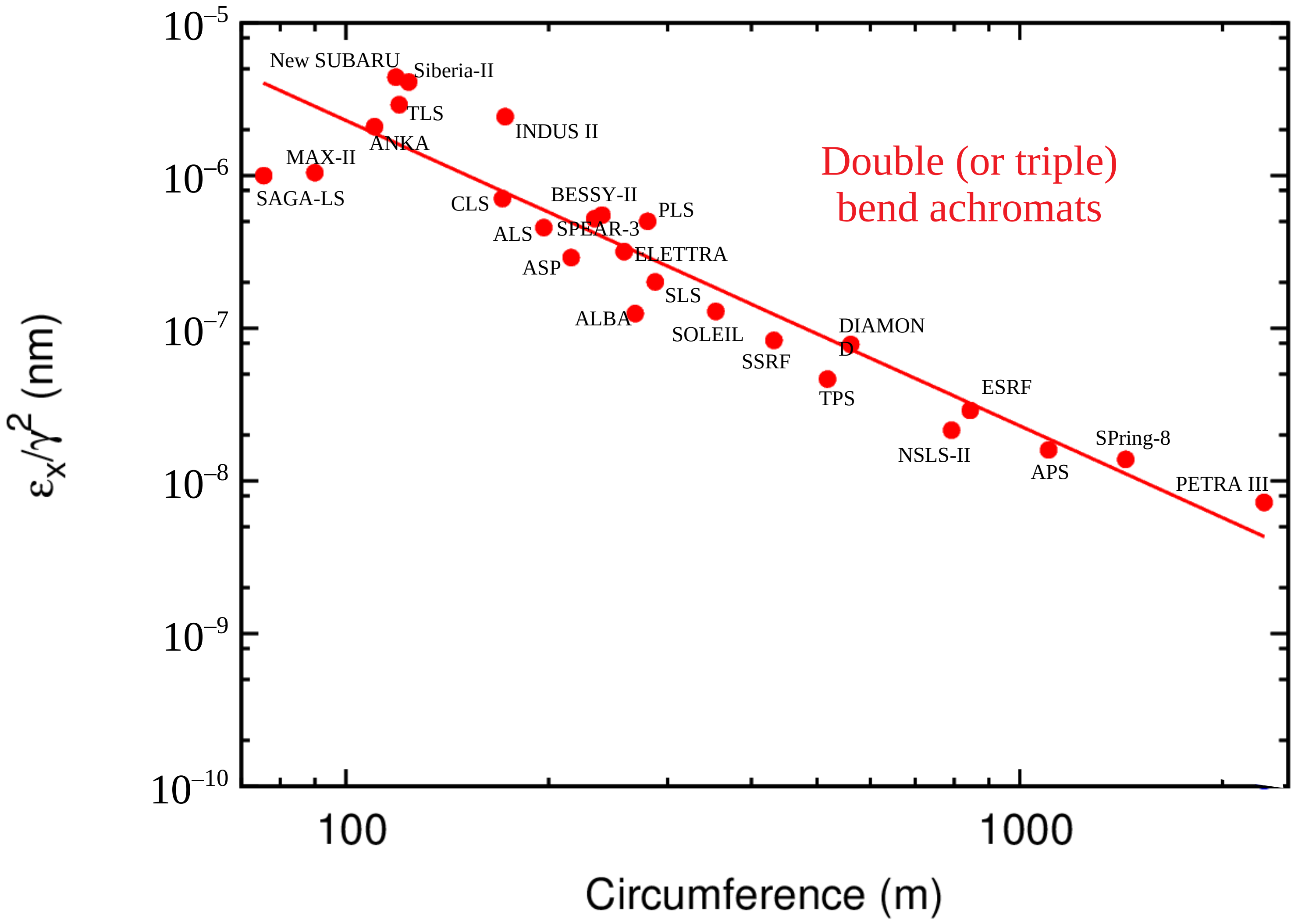


We will discuss our efforts to understand and predict collective electron dynamics in high-brightness storage rings that try to maximize electron beam current while minimizing emittance.

Upgrade projects are underway around the world



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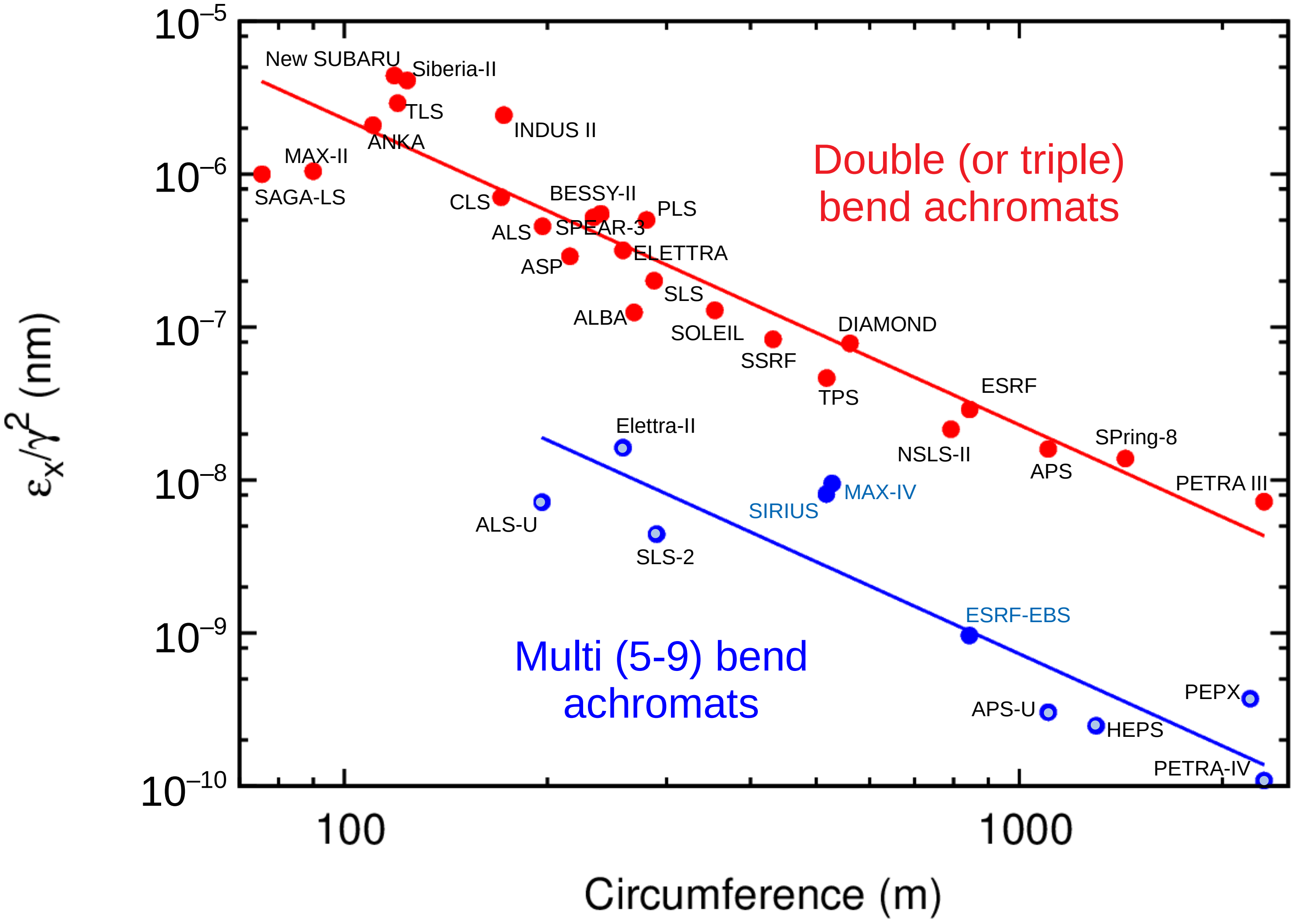
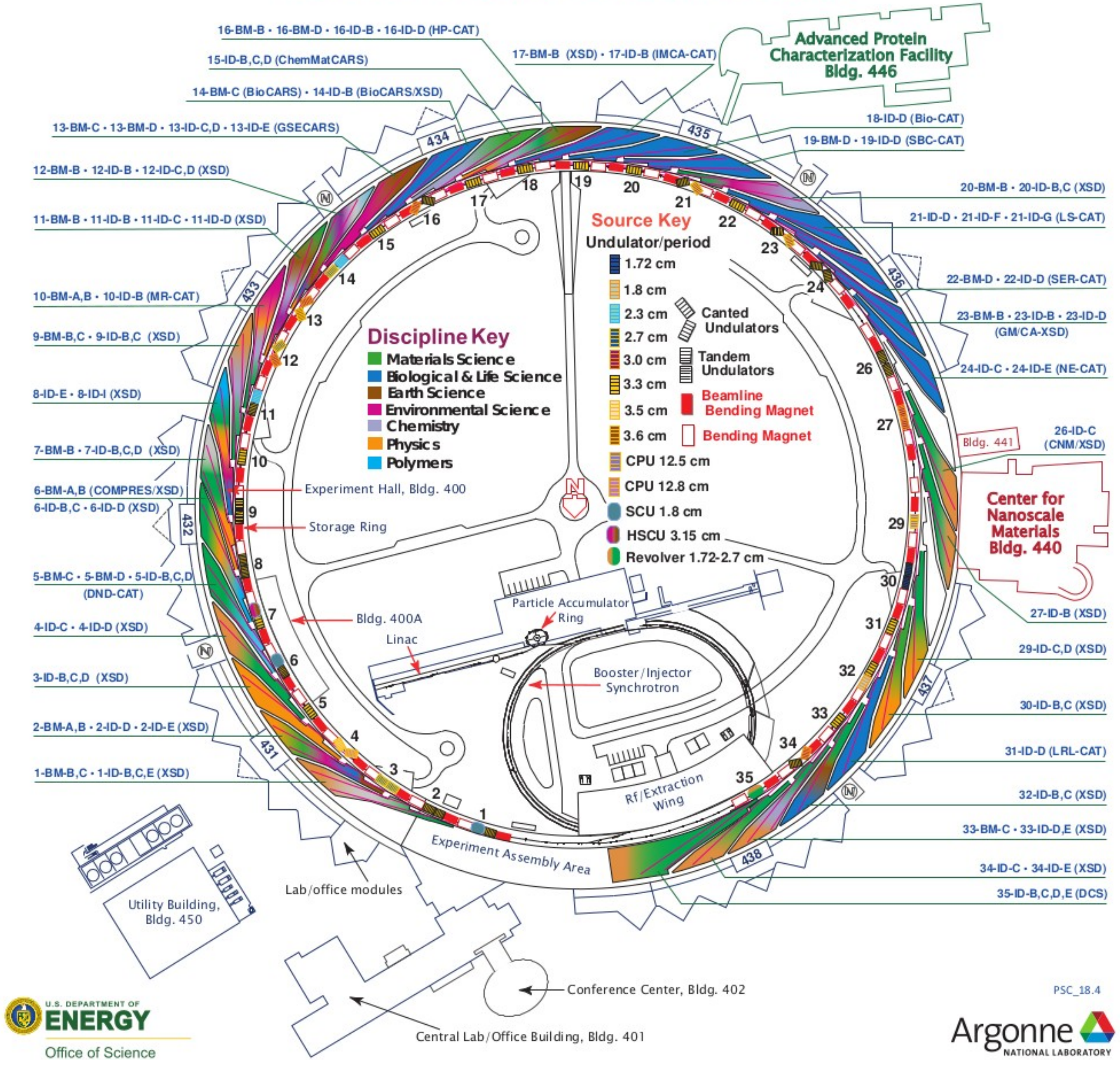
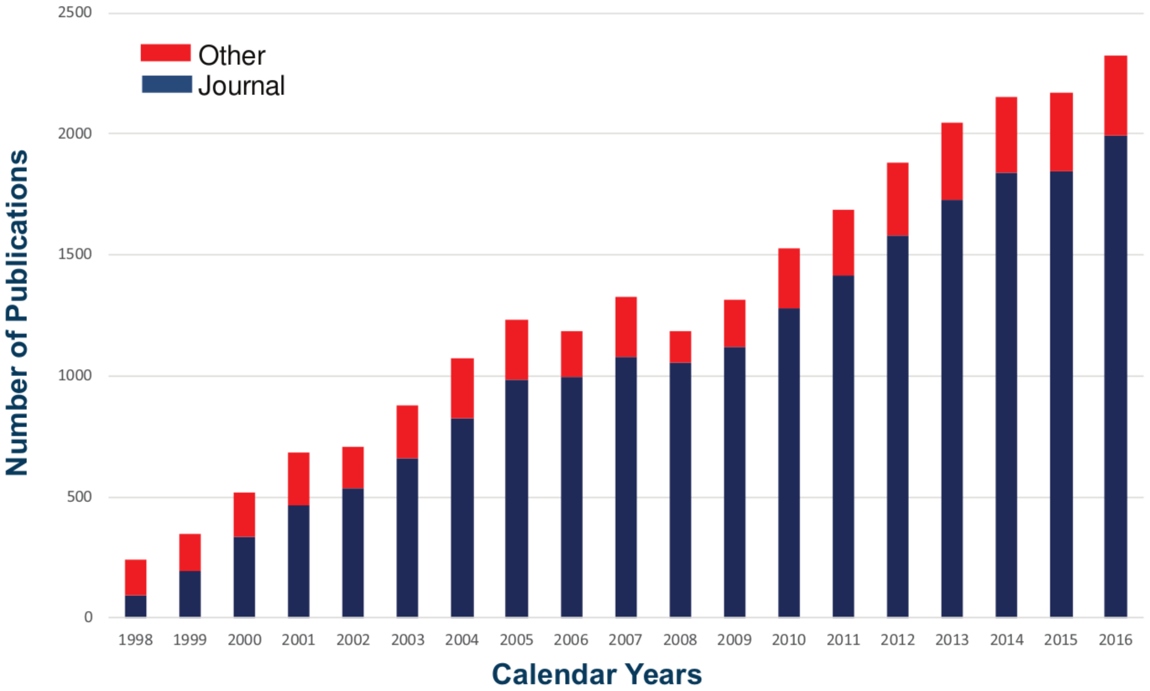


APS	→	APS-Upgrade
I_{total} = 100 mA	→	I_{total} = 200 mA
ε_x = 2.5 nm	→	ε_x = 40 pm

X-ray brightness increases by ~two orders of magnitude

Advanced Photon Source – facility view

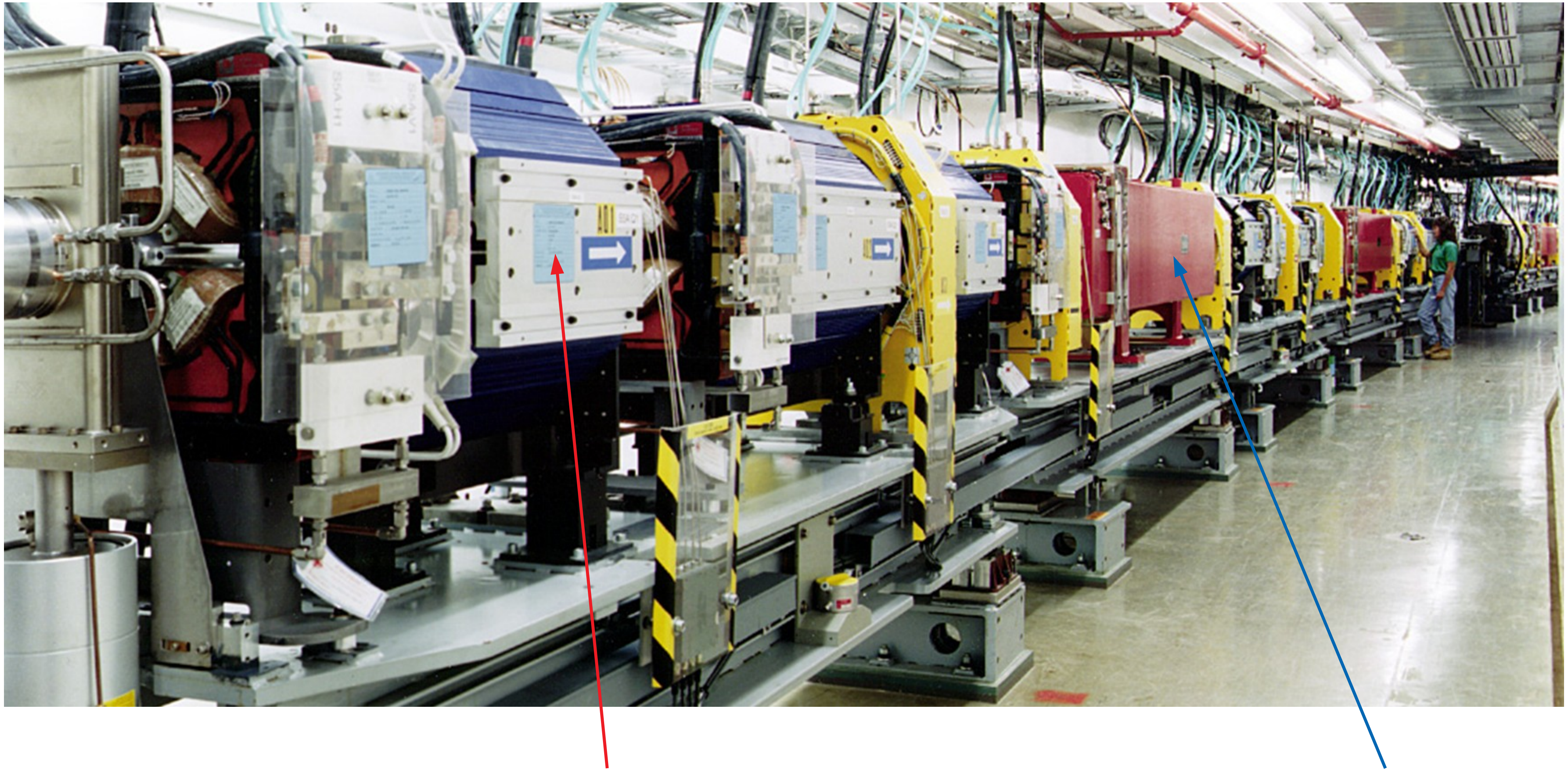




To search lists of APS publications see https://beam.aps.anl.gov/pls/apsweb/pub_v2_0006.review_start_page

FROM: APS Science 2018 V1, ANL-18/40

Advanced Photon Source – Operations/Engineering view

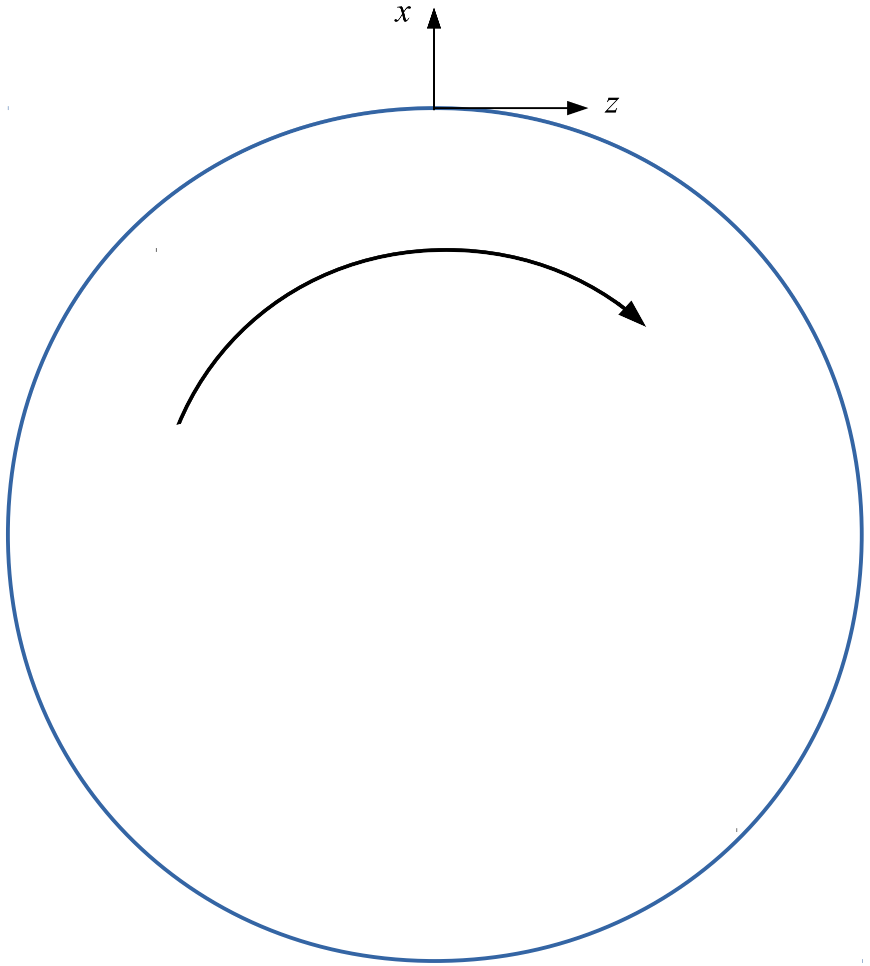


Quadrupole magnet

Dipole magnet

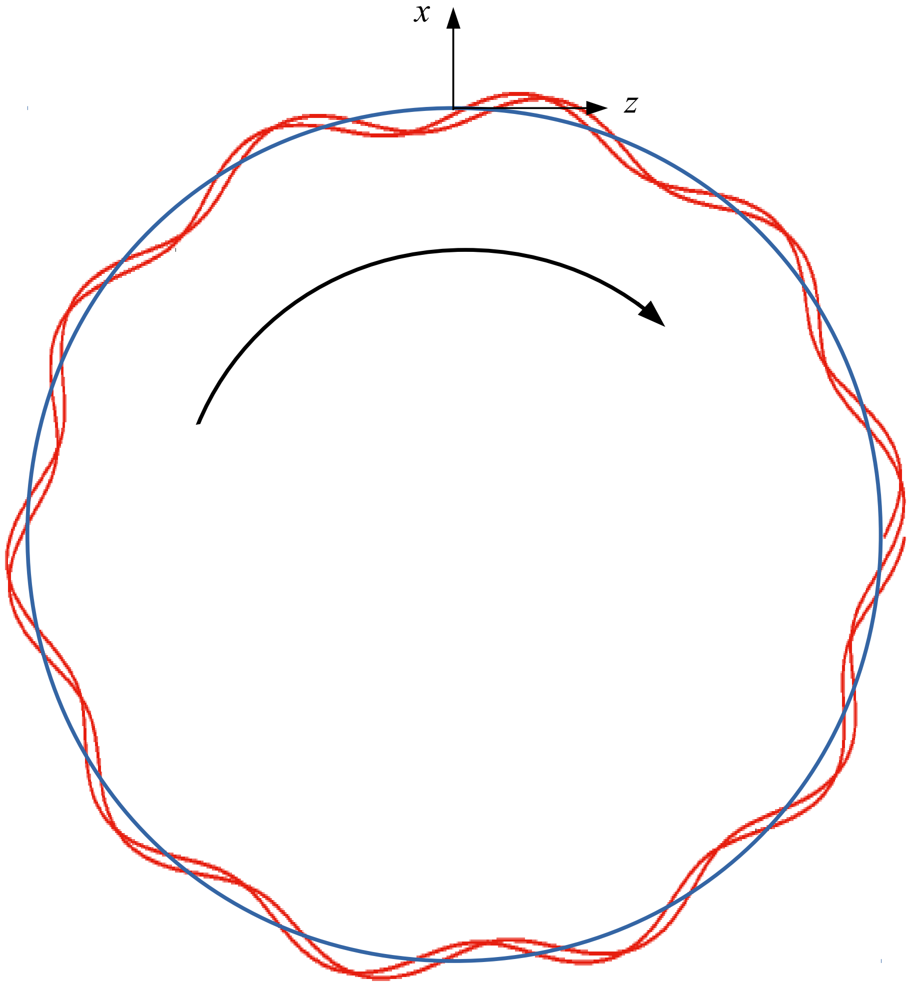
High brightness storage ring – Simplified physics picture

- Bending dipole magnets define a closed reference trajectory with revolution period T_0



High brightness storage ring – Simplified physics picture

- Bending dipole magnets define a closed reference trajectory with revolution period T_0
- Small amplitude motion in the vertical and horizontal directions is approximately harmonic
  - Sequence of quadrupole magnets provide a quadratic potential in the transverse plane.
  - Particle makes many (>10) transverse betatron oscillations during one round trip in the ring.
  - Sextupole magnets control how the transverse betatron frequency depends upon the energy.



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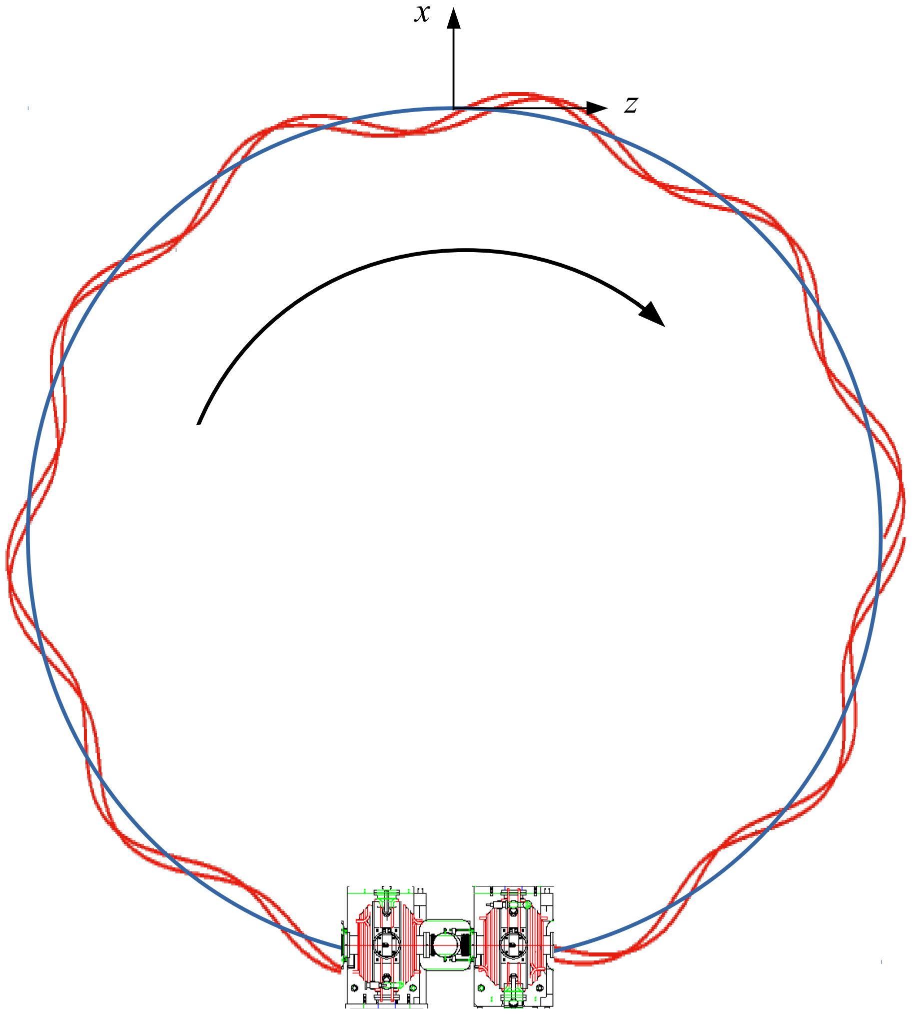

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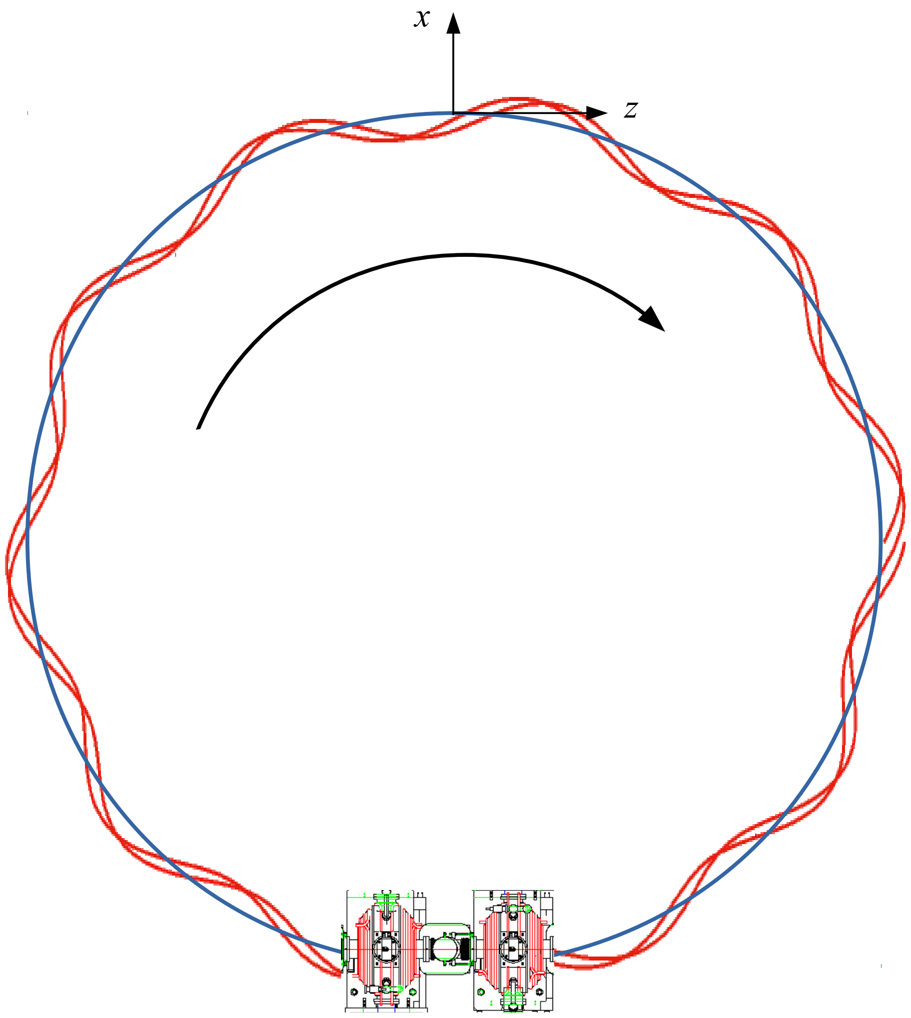
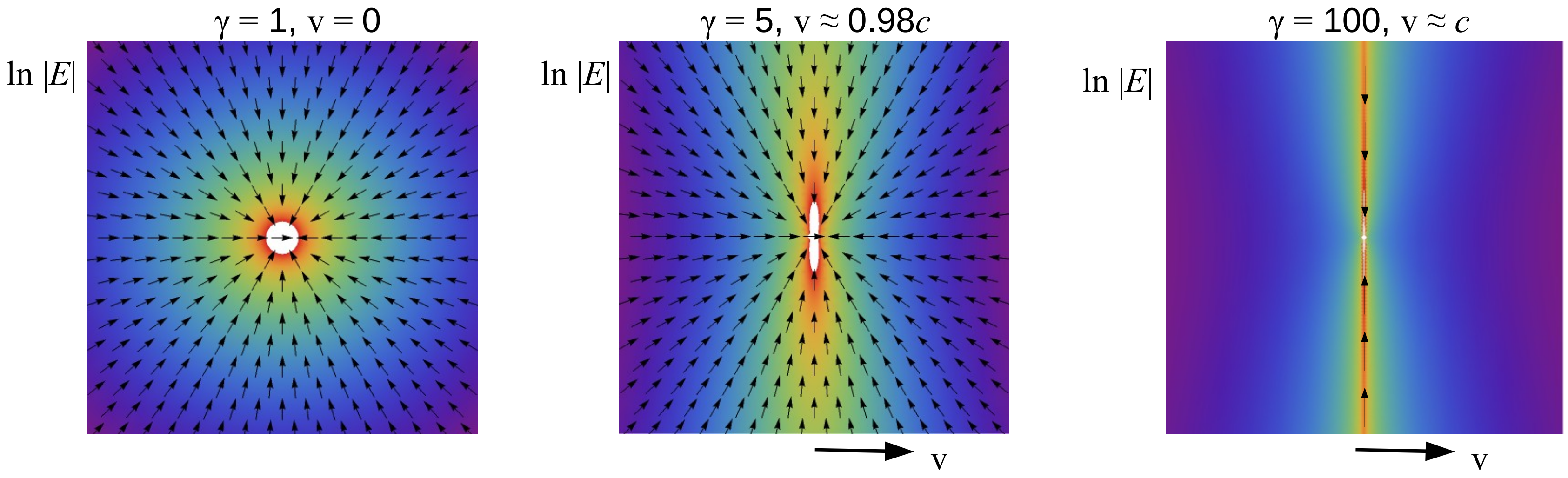
- The equilibrium is set by the balance of radiation damping and stochastic diffusion of photon emission



Coulomb interaction due to direct space charge is weak

- The Coulomb field of a relativistic particle with kinetic energy γmc^2 becomes compressed into the angle $\sim 1/\gamma$.



1 GeV electrons: $\gamma \sim 2{\times}10^3$

The APS-U will be at 6 GeV, and we have $\gamma \sim 12{\times}10^3$

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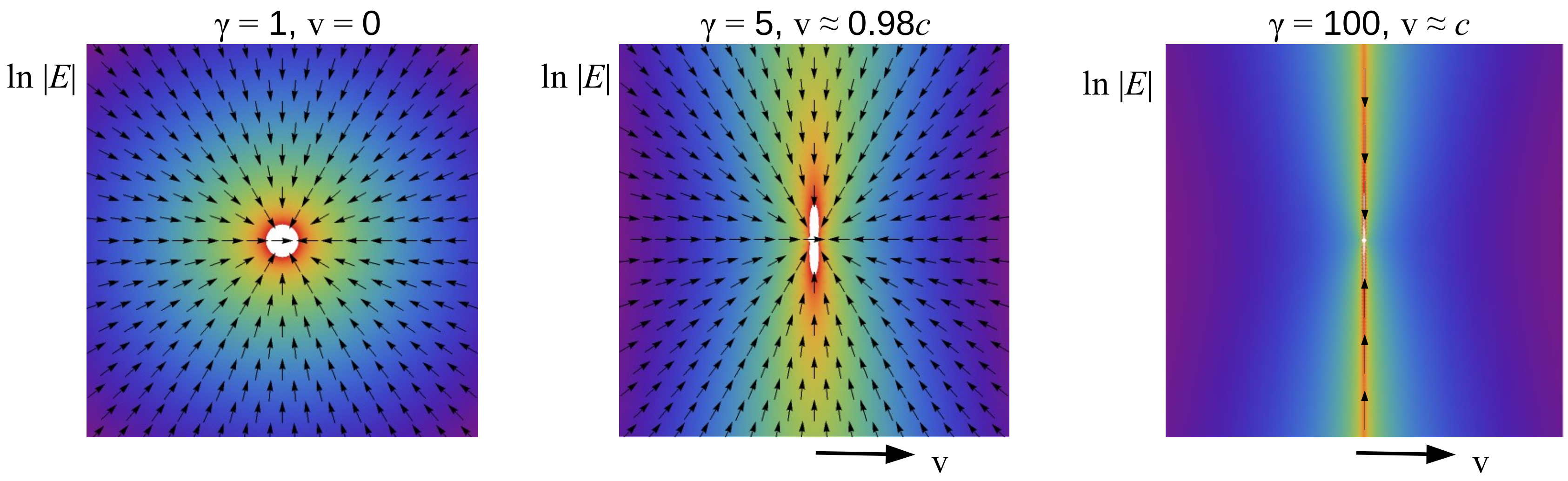

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- In the ultra-relativistic ($\gamma \to \infty$) limit the Coulomb field becomes a pancake
  - Longitudinal electric force goes to zero as $1/\gamma^2$
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- An ultra-relativistic beam in a perfectly conducting chamber with constant cross-section has no collective forces

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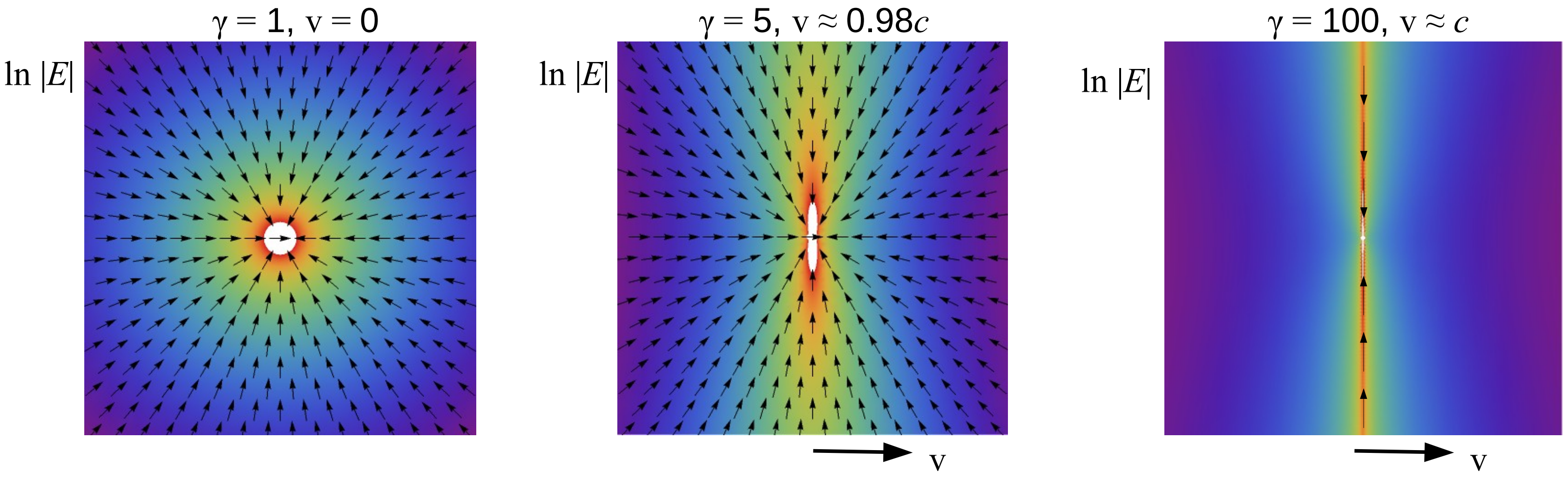

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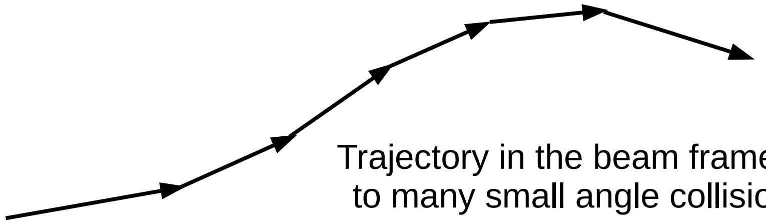
For our parameters, the finite γ mostly affects the equilibrium.

- An ultra-relativistic beam in a perfectly conducting chamber with constant cross-section has no collective forces

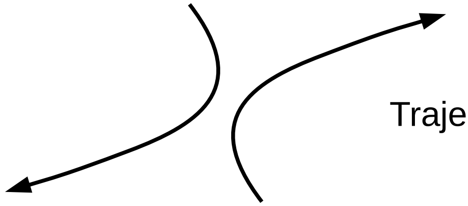
Chambers with finite resistivity and whose cross section varies can drive important collective forces.

While weak, Coulomb scattering impacts the equilibrium

- Many small angle scattering events can lead to a slow growth in emittance (intrabeam scattering)[1,2]
  - Growth rate depends upon particle density
  - Equilibrium is reached when the growth rate is matched by damping due to synchrotron emission
- Large angle scattering can lead to particle loss[3]
  - Large angle scattering can transfer a significant fraction of the transverse momentum to the longitudinal plane
  - These "off-momentum" particles are lost when $\Delta p_z/p_0$ ~ few %
  - The resulting Touschek loss rate limits the lifetime of the beam



Trajectory in the beam frame due to many small angle collisions.



Trajectories in the beam frame of a large angle collision.

[1] A. Piwinski, Intra-beam scattering, Proc. 9th Int. Conf. on High Energy Accel.m p. 405 (1974).
[2] J.D. Bjorken and S.K. Mtingwa, Intrabeam scattering, Particle Accel. 13 115 (1983).
[3] A. Piwinski, The Touschek effect in strong focusing storage rings, DESY-98-179, Hamburg, Germany (1998).

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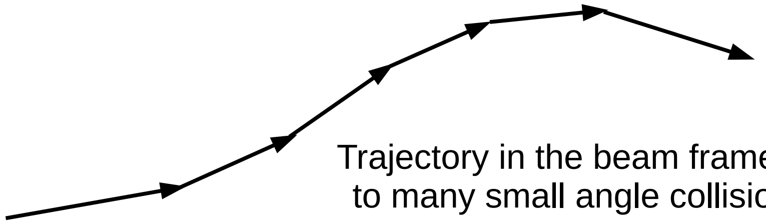
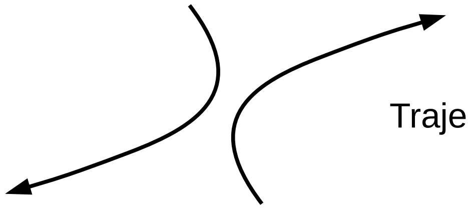
  - These “off-momentum” particles are lost when $\Delta p_z/p_0$ ~ few %
  - The resulting Touschek loss rate limits the lifetime of the beam
- These effects can be controlled by reducing beam density
  - Use “round” electron beams that have equal emittances in the two planes: $\varepsilon_y = \varepsilon_x$.
  - Use long electron beams that reduce peak current.



Trajectory in the beam frame due to many small angle collisions.



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While weak, Coulomb scattering impacts the equilibrium

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  - The resulting Touschek loss rate limits the lifetime of the beam
- These effects can be controlled by reducing beam density
  - Use "round" electron beams that have equal emittances in the two planes: $\varepsilon_y = \varepsilon_x$.
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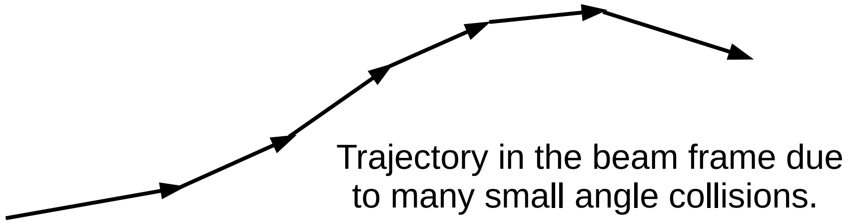
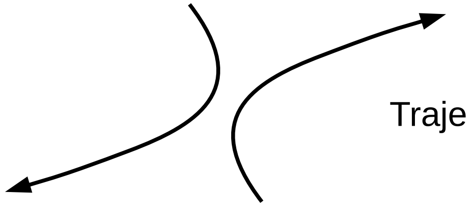
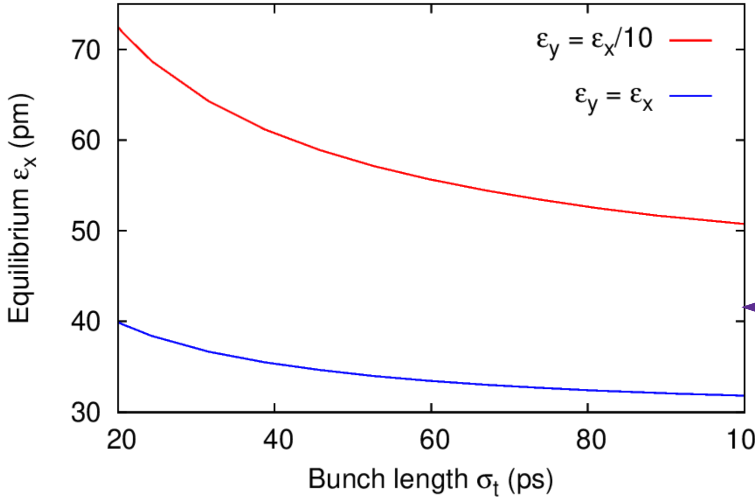


Trajectory in the beam frame due to many small angle collisions.



Trajectories in the beam frame of a large angle collision.

Impact of intrabeam scattering on APS-U emittance



Ideal $\varepsilon_x = 42$ pm if $\varepsilon_y = \varepsilon_x/10$.

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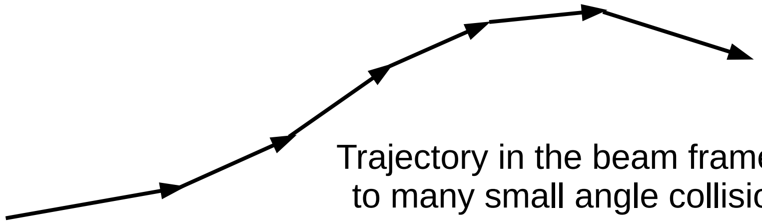

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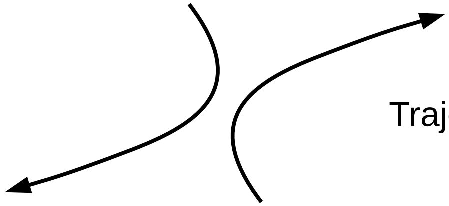

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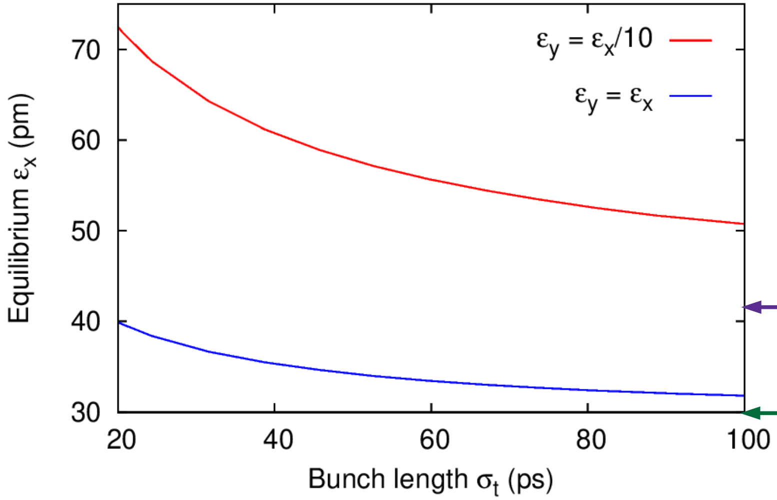

Ideal $\varepsilon_x = 42$ pm if $\varepsilon_y = \varepsilon_x/10$.

Ideal $\varepsilon_x = 30$ pm if $\varepsilon_y = \varepsilon_x$.

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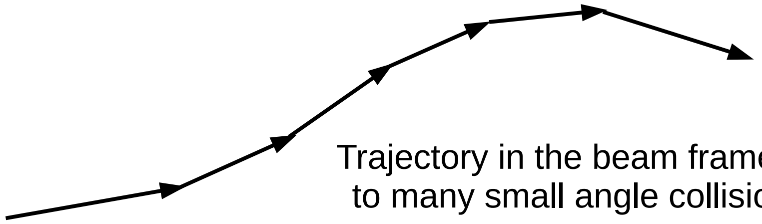
- The increase in electron beam lifetime obtained with round, long electron bunches is perhaps even more significant



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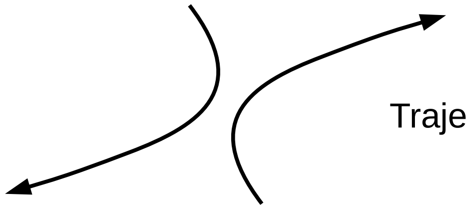

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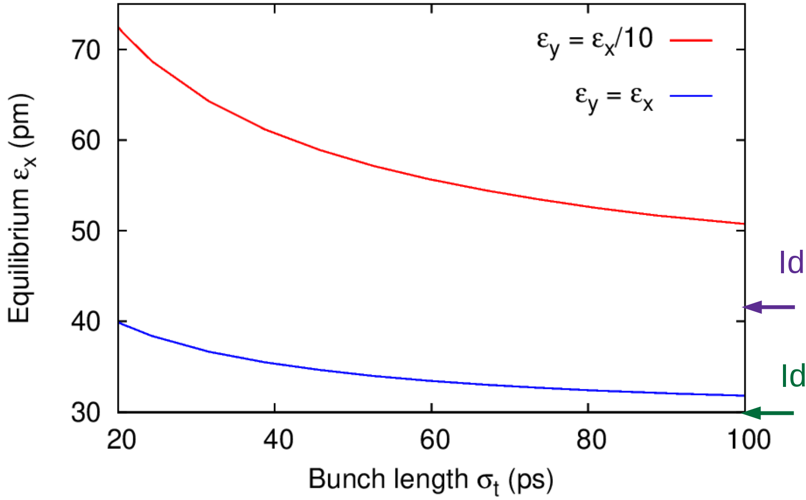

Ideal ε_x = 41 pm if $\varepsilon_y = \varepsilon_x/10$.

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We can make a round beam by exploiting resonant coupling between the horizontal and vertical motion

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- We describe the dynamics at turn T by as coupled oscillators[4]:

Horizontal SHO: $$\frac{du_x}{dT} - \frac{i}{2}\{\omega_x - \omega_y\}u_x = \frac{i\kappa}{2}u_y$$

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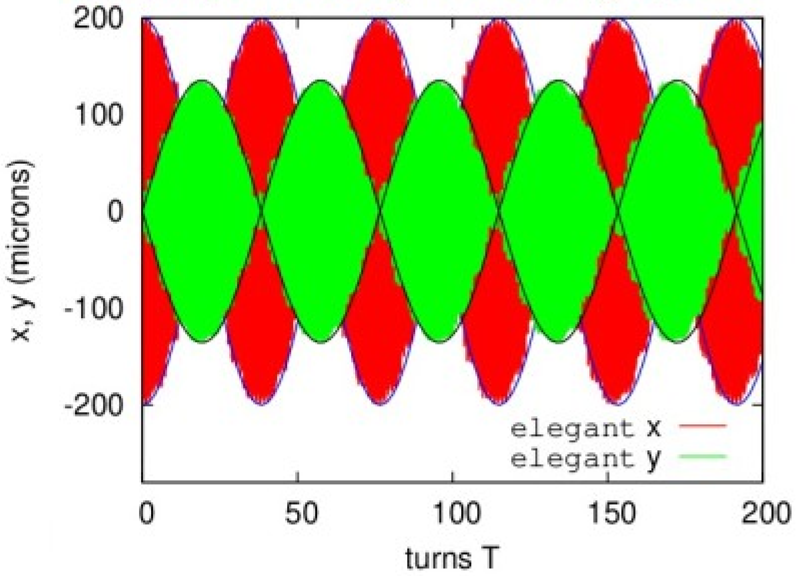
- Initial oscillation along x becomes oscillation along y

Single particle dynamics when $\{\omega_x - \omega_y\} = 0$



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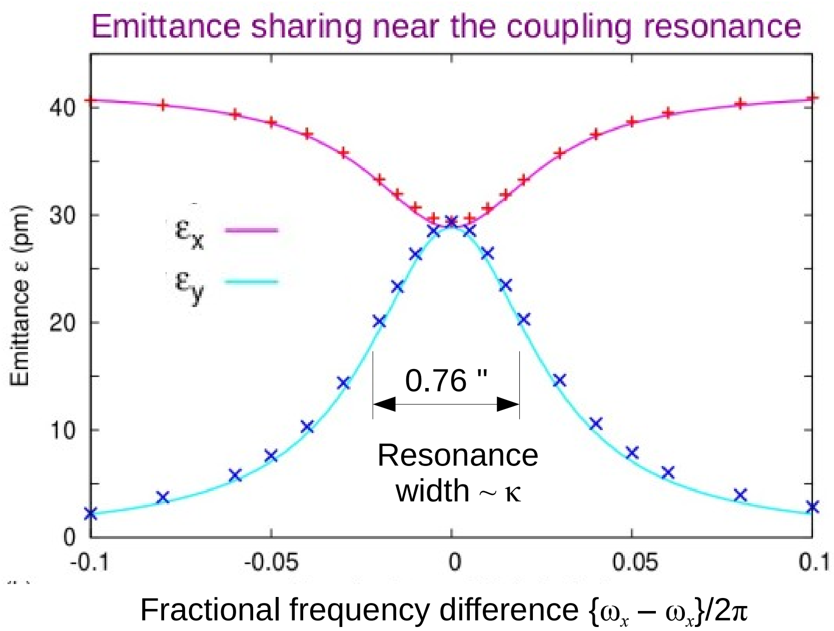

- Initial oscillation along x becomes oscillation along y
- Independent degrees of freedom are combinations of u_x and u_y
- These single particle dynamics will influence collective stability

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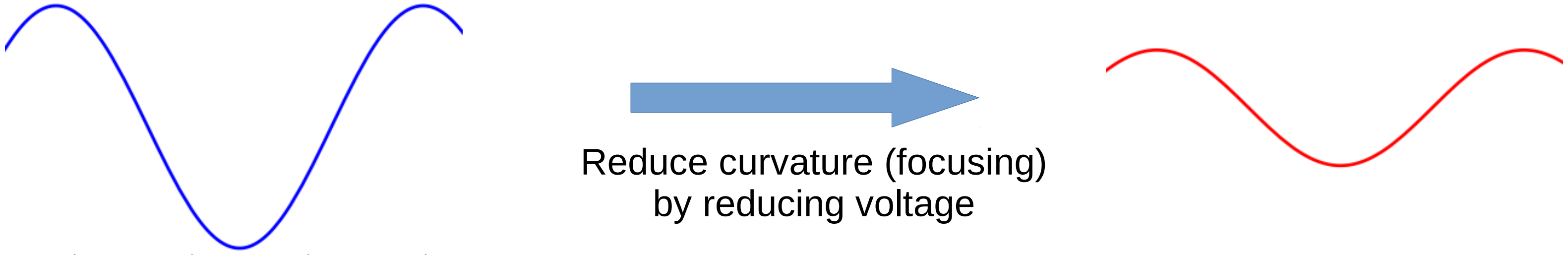
We stretch the bunch length by adding another rf system

- Rf cavities accelerate and confine particles in a potential that is sinusoidal in time
- We could increase the bunch length by changing the voltage



Reduce curvature (focusing)
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Results in a potential well that
in too shallow to trap all the
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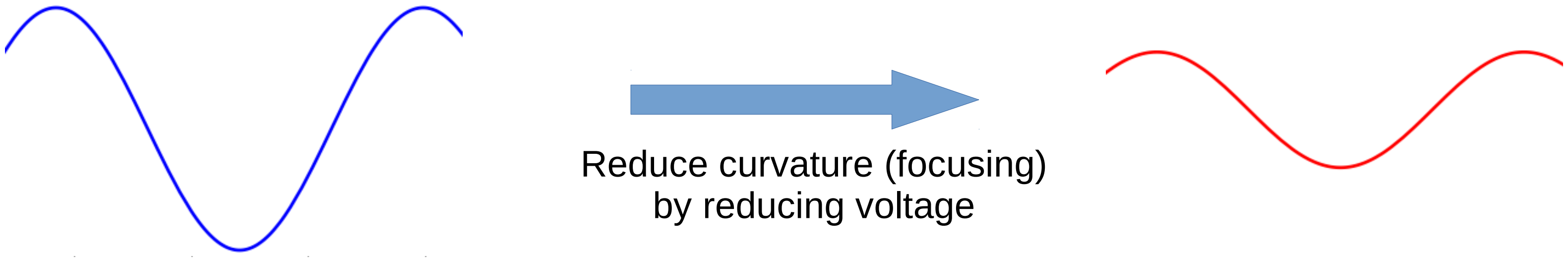

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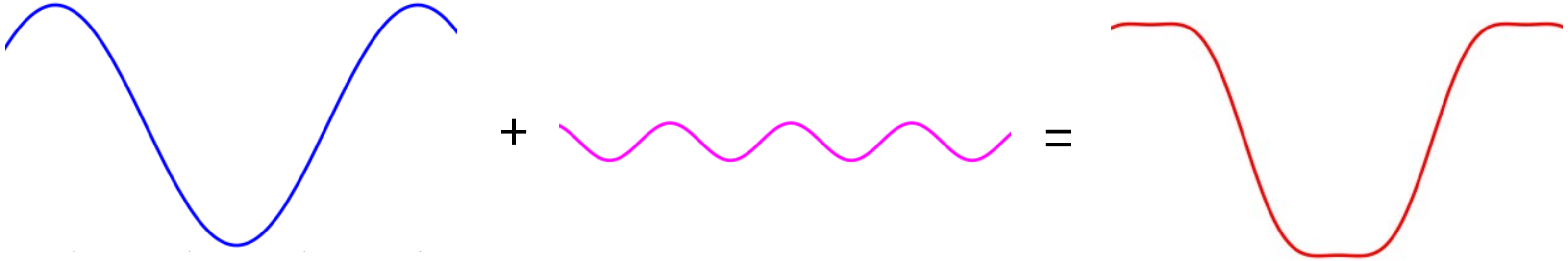

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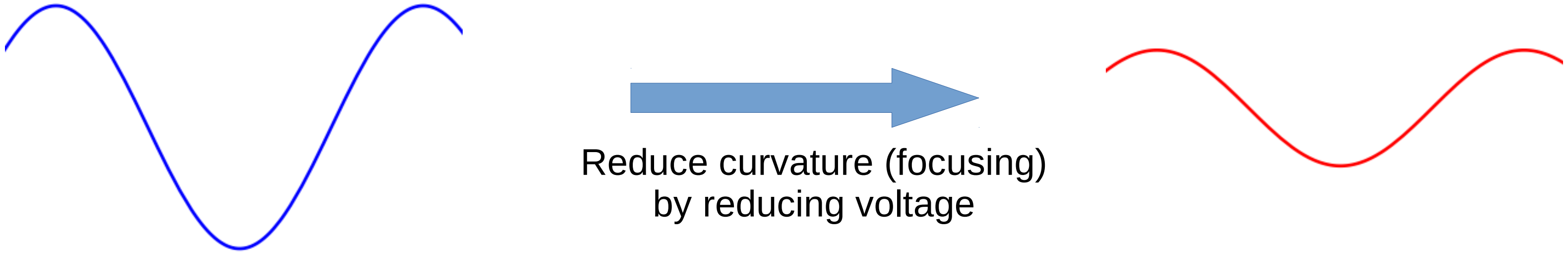

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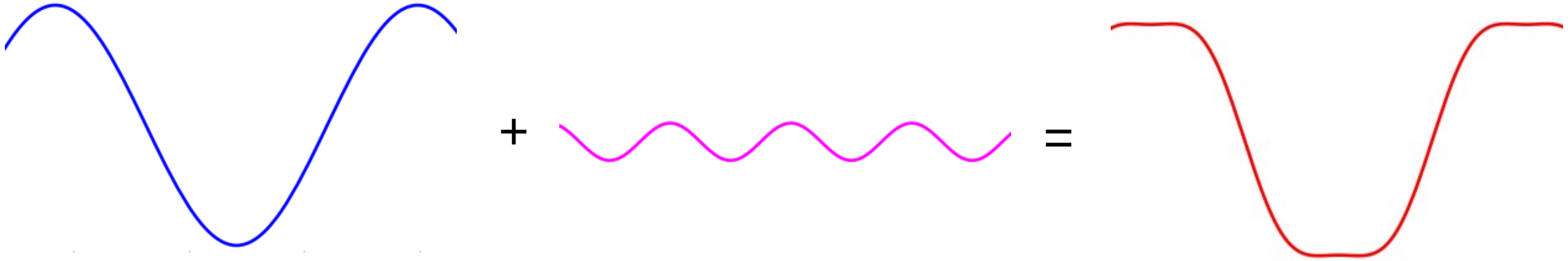

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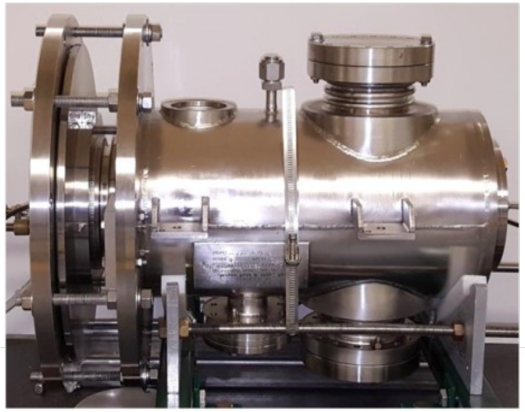
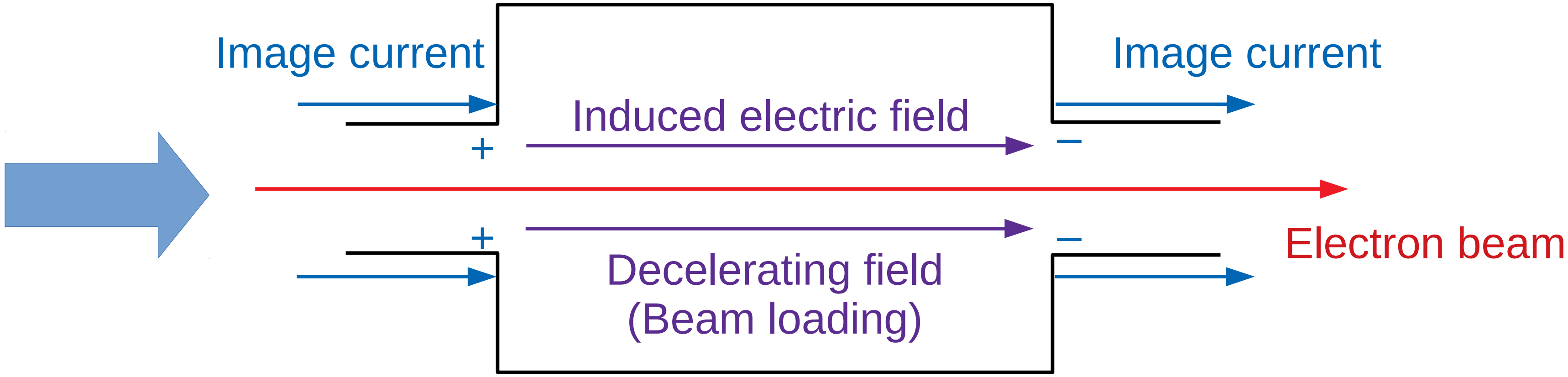
- The APS-U will add a superconducting rf cavity at the 4th harmonic



The electron beam can be used to supply the harmonic voltage for bunch lengthening

- Accelerating rf cavities powered at 352 MHz by klystrons (soon to be solid state amplifiers)
- We plan to have the bunch lengthening (harmonic) cavity get its voltage from the electron beam itself



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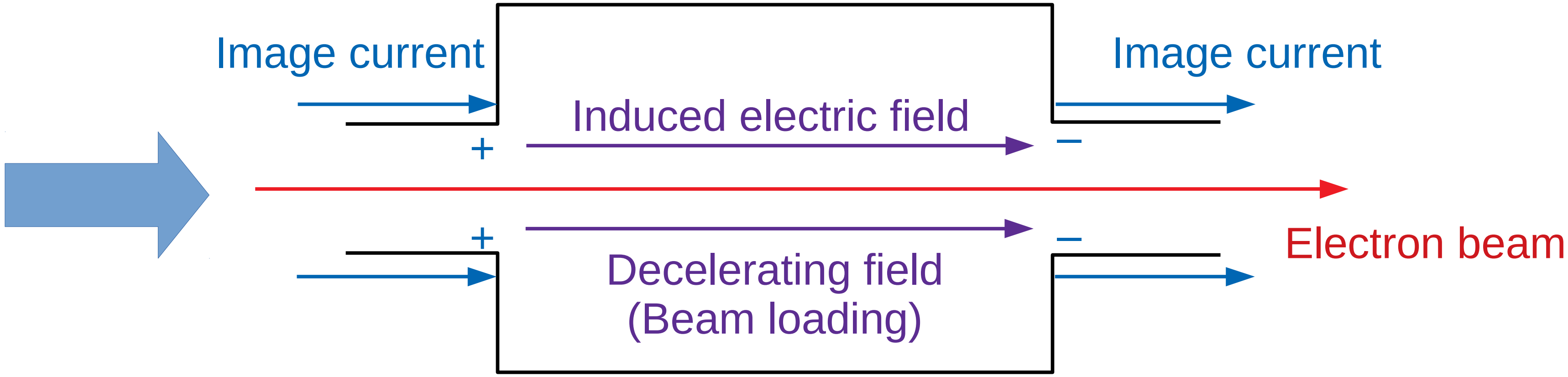

Image current

Image current

Induced electric field

Electron beam

Decelerating field
(Beam loading)

$$[\text{Cavity Voltage}] = \underbrace{[\text{Beam current}]} \times \underbrace{[\text{Cavity Impedance } Z_c(\omega)]}$$

Beam drives the cavity at frequencies that are harmonics of the revolution frequency

Cavity impedance is well-described by a simple resonator

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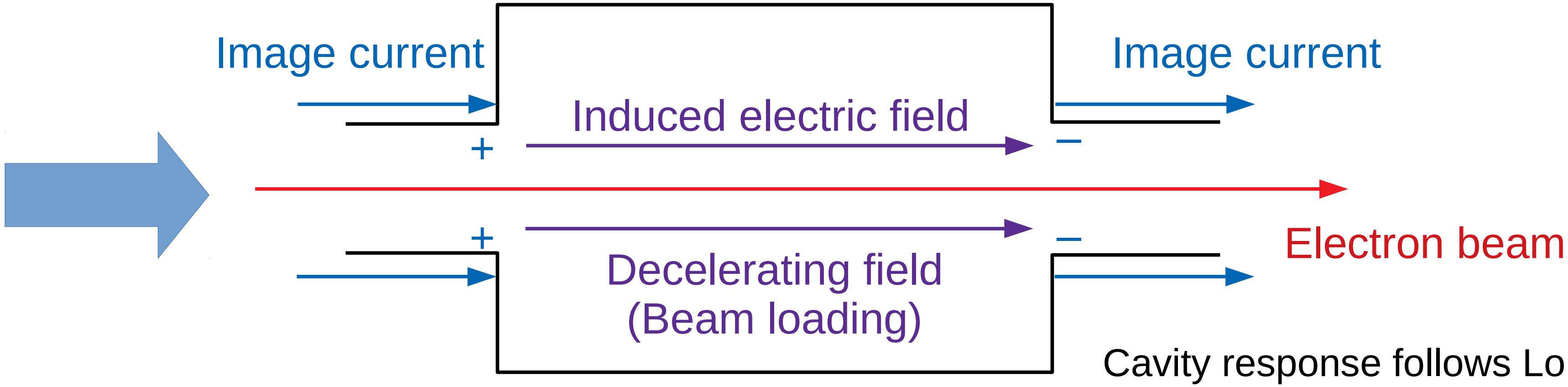

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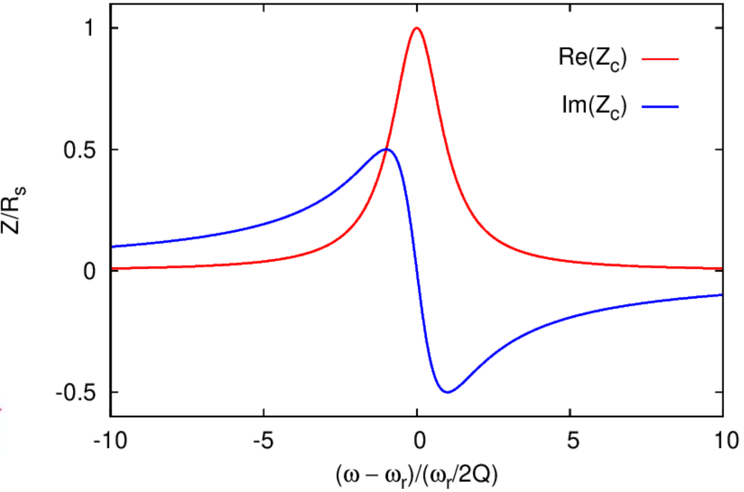
$$Z_c(\omega) = R_s \frac{1 - i(\omega - \omega_r)/2Q}{1 + [(\omega - \omega_r)/2Q]^2}$$

Cavity response follows Lorentzian

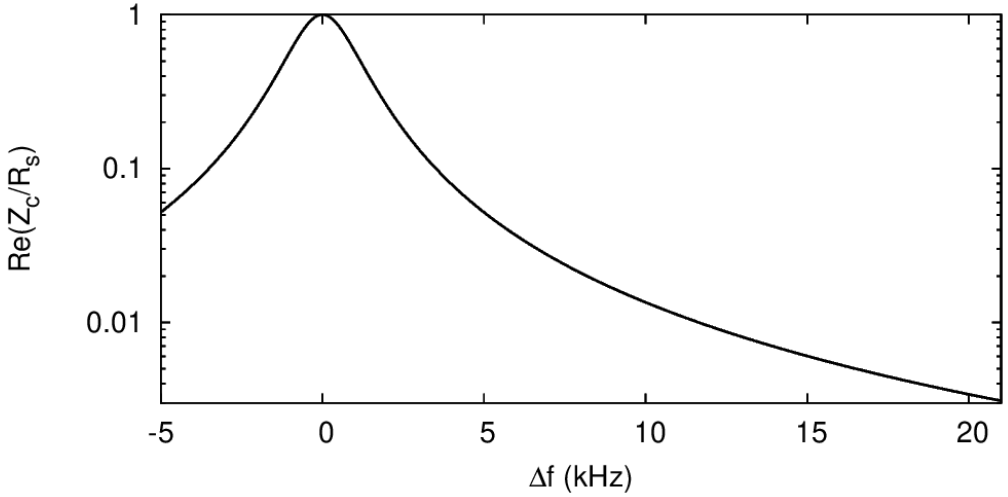


Passive harmonic cavity significantly lengthens bunch

- Electron beam drives the cavity at multiples of the revolution frequency in the ring

Single electron with revolution time T_0 $$I(t) = e\sum_n \delta(t - nT_0) \;\Rightarrow\; I(\omega) = \frac{e}{T_0}\sum_n \delta(\omega - 2\pi n/T_0)$$

- Beam loading voltage is controlled by tuning the resonant cavity frequency close to a multiple of the revolution frequency



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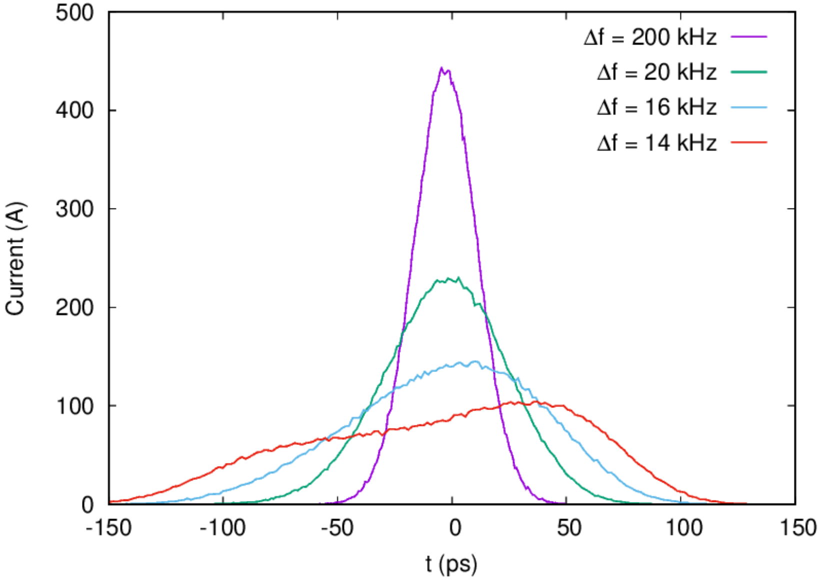

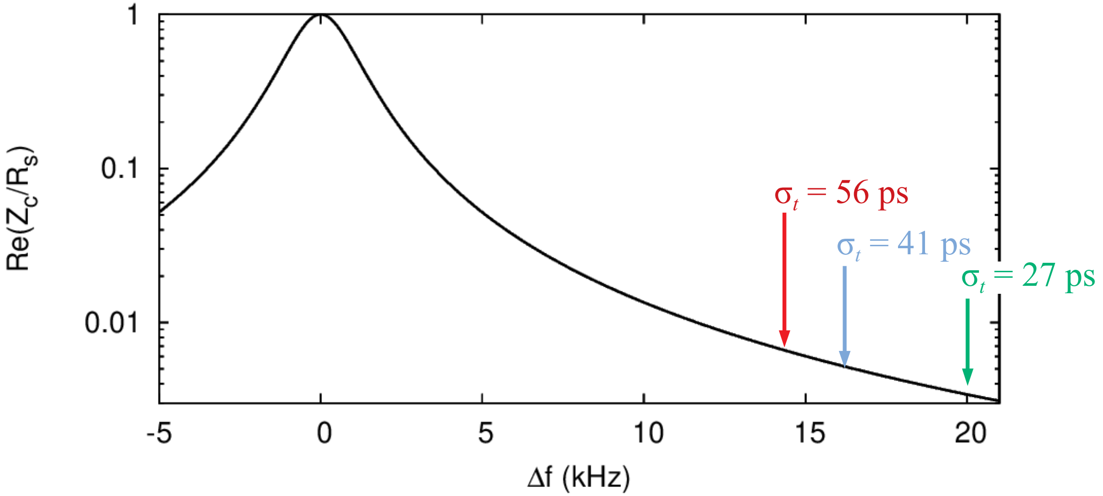

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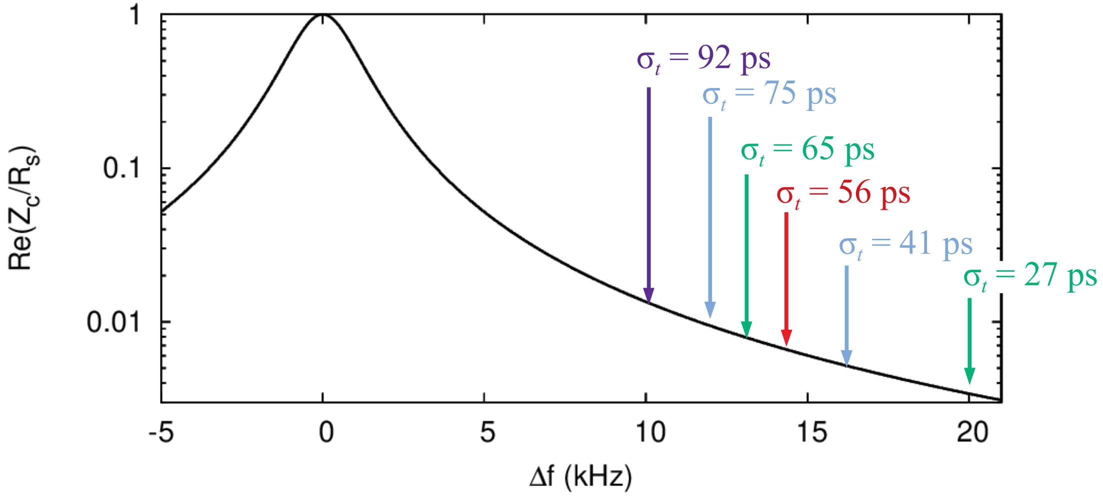

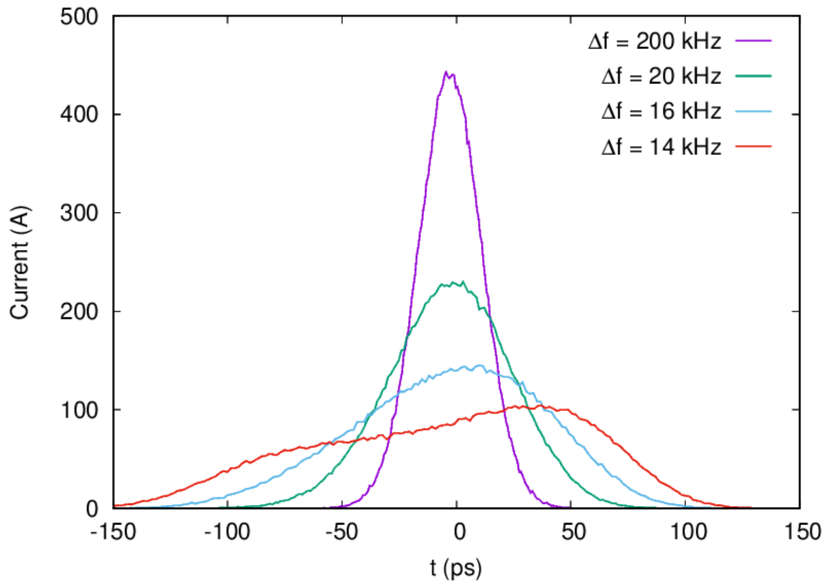

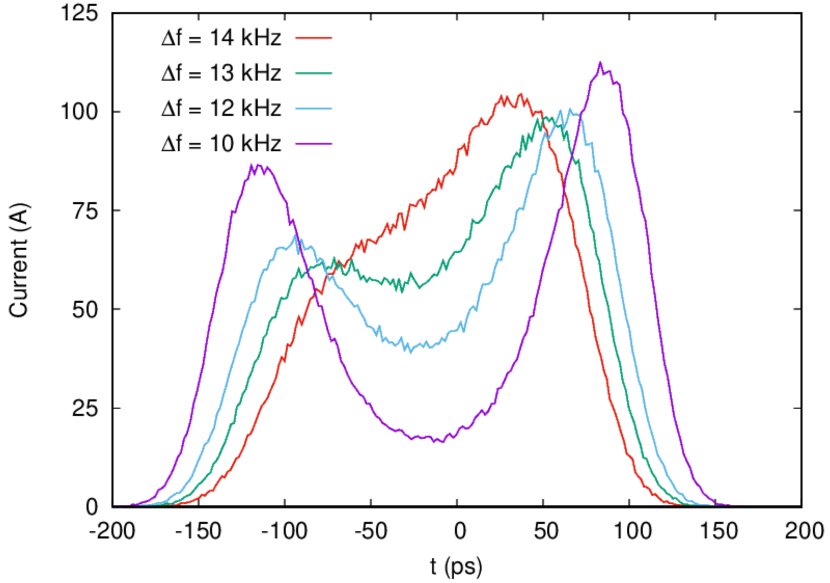

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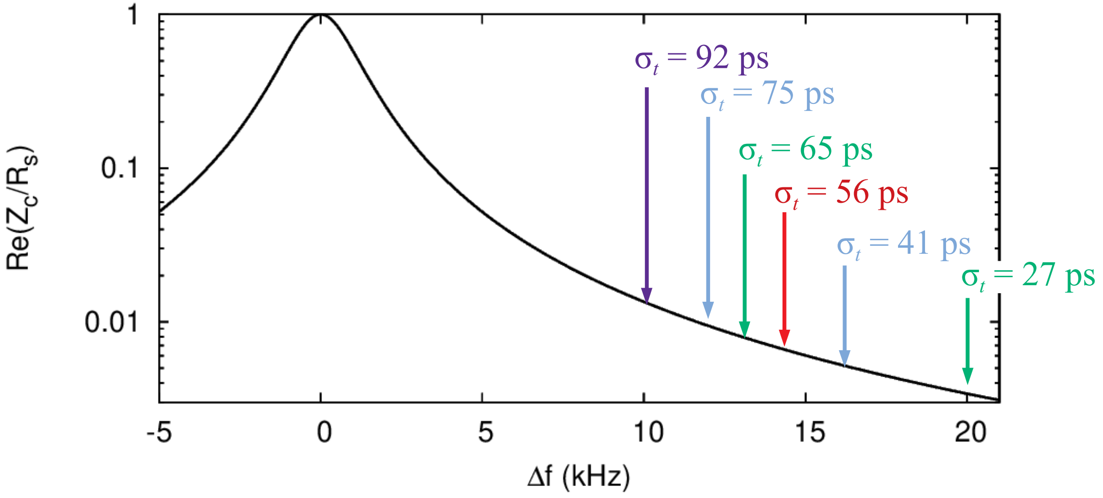

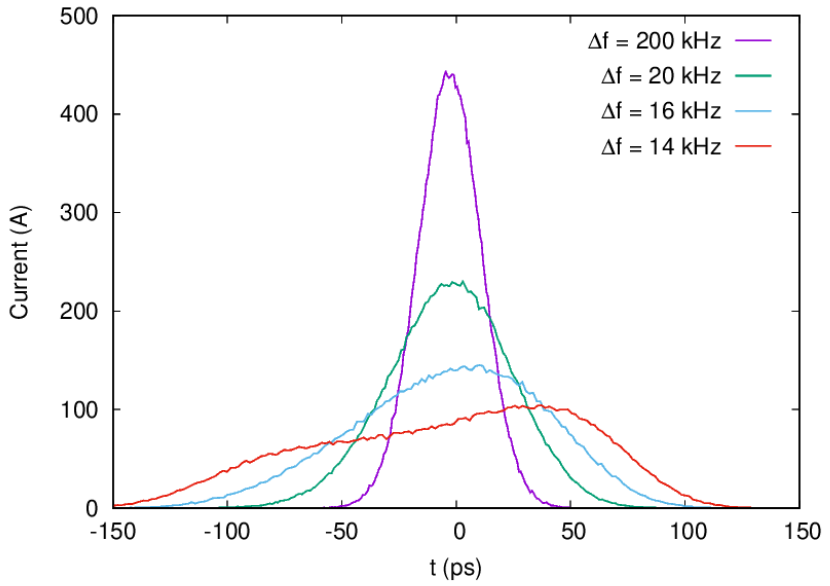

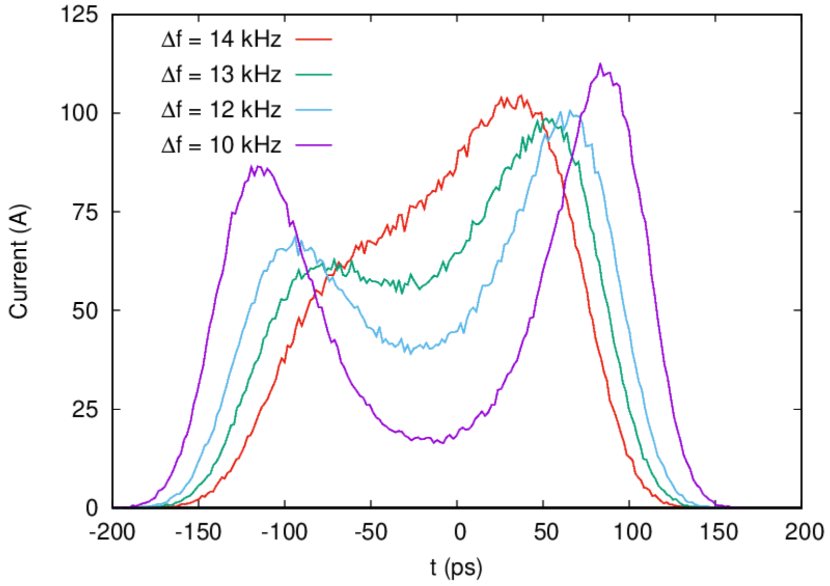

- We want to minimize effects of Coulomb collisions that lead to emittance growth (small angle) and particle loss (large angle)
  - This amounts to minimizing $\int dt\; I(t)^2 \;\rightarrow\; \Delta f = 10$ kHz
- Longitudinal motion is nonlinear with small characteristic frequency

Beam loading in rf cavities can also drive instabilities

- Electron beam drives voltage in all rf cavities at harmonics of the revolution frequency
  - Beam loading in harmonic cavity gives bunch lengthening
  - Beam loading in main cavities can distort the focusing field and affect stability
- Exciting higher order modes in the accelerating cavities can lead to instability

[5] K. A. Thompson and R. D. Ruth, Transverse and longitudinal coupled-bunch instabilities in trains of closely spaced bunches, SLAC Report No. SLAC-PUB-4872 (1989).
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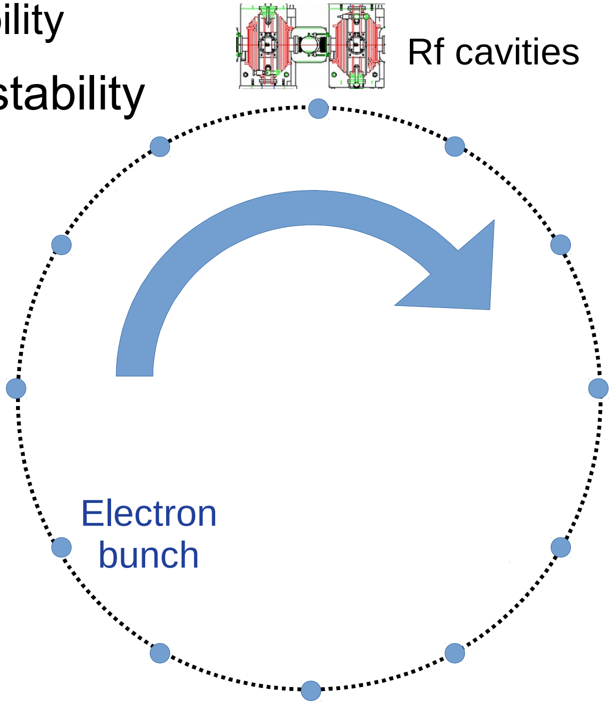
- Simplest model for N bunches in the ring is of N harmonic oscillators that are coupled together by the beam induced voltage[5]
  - Normal modes are found by diagonalizing the system
  - Complex frequencies indicate damped or growing perturbations
- More accurate model includes the nonlinear longitudinal potential[6]



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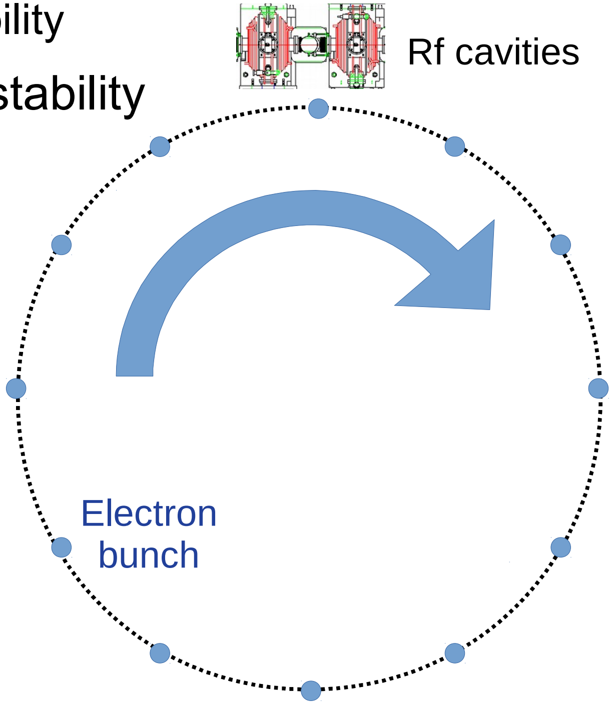
- For our system, the frequencies Ω associated with the motion of the bunch centroids $\langle z \rangle_n$ are given by[7]

$$\langle z \rangle_n = \sum_{j=0}^{N} \mathsf{M}_{n,j} \langle z \rangle_j \mathscr{D}(\Omega)$$

Linear coupling matrix that can be diagonalized to obtain normal modes



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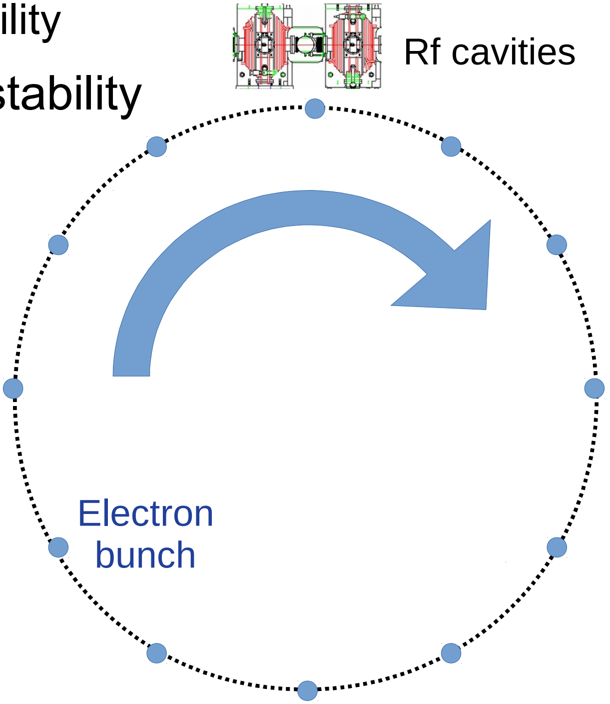
[7] R. R. Lindberg, “Theory of coupled-bunch longitudinal instabilities in a storage ring for arbitrary rf potentials,” Phys. Rev. Accel. Beams **21**, 124402 (2018)

Beam loading in rf cavities can also drive instabilities

- Electron beam drives voltage in all rf cavities at harmonics of the revolution frequency
  - Beam loading in harmonic cavity gives bunch lengthening
  - Beam loading in main cavities can distort the focusing field and affect stability
- Exciting higher order modes in the accelerating cavities can lead to instability
- Simplest model for N bunches in the ring is of N harmonic oscillators that are coupled together by the beam induced voltage[5]
  - Normal modes are found by diagonalizing the system
  - Complex frequencies indicate damped or growing perturbations
- More accurate model includes the nonlinear longitudinal potential[6]
- For our system, the frequencies Ω associated with the motion of the bunch centroids $\langle z\rangle_n$ are given by[7]



$$\langle z\rangle_n = \sum_{j=0}^{N} \mathrm{M}_{n,j}\langle z\rangle_j \mathscr{D}(\Omega)$$

Nonlinear dispersion relation for complex frequency Ω

Linear coupling matrix that can be diagonalized to obtain normal modes

$$\mathscr{D}(\Omega) \sim \int dx\ \bar{F}(x)\frac{g(x)}{\Omega-\omega(x)}$$

Equilibrium distribution $\bar{F}$

Singularities associated with the nonlinear frequency $\omega(x)$

→ Landau damping

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(In)stability in the presence of Landau damping

- Landau damping reduces the amplitude of a coherent oscillation in a loss-less system
  - Particles whose nonlinear frequency is close to the coherent frequency interact strongly with wave
  - If there are more particles taking energy from the wave then receiving it, then the wave is damped
- Mathematically, Landau damping results from the resonant denominator

$$\mathscr{D}(\Omega) \sim \int dx\ \bar{F}(x) \frac{g(x)}{\Omega - \omega(x)}$$

This integral is discontinuous when the imaginary part of Ω changes sign, since the Sokhotski-Plemeli theorem states

$$\lim_{\epsilon \to 0} \int_a^b dx\, \frac{f(x)}{x \mp i\epsilon} = \mathcal{P} \int_a^b dx\, \frac{f(x)}{x} \pm i\pi f(0) \quad \text{for } a < 0 < b.$$

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- When Im(Ω) < 0 we must analytically continue the dispersion relation → Landau damping

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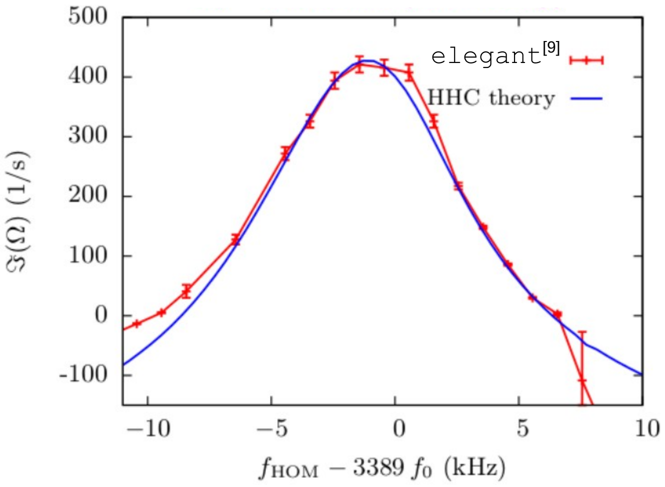
- Landau[8] showed that the dispersion relation in red only applies when Im(Ω) > 0
- When Im(Ω) < 0 we must analytically continue the dispersion relation → Landau damping
- Resulting theoretical predictions agree with simulations
- Unfortunately, Landau damping doesn't rescue us
  - The growth rates are large and radiation damping is weak
  - The oscillation frequency is small → Landau damping rate is small
  - There are many resonant modes that contribute to instability



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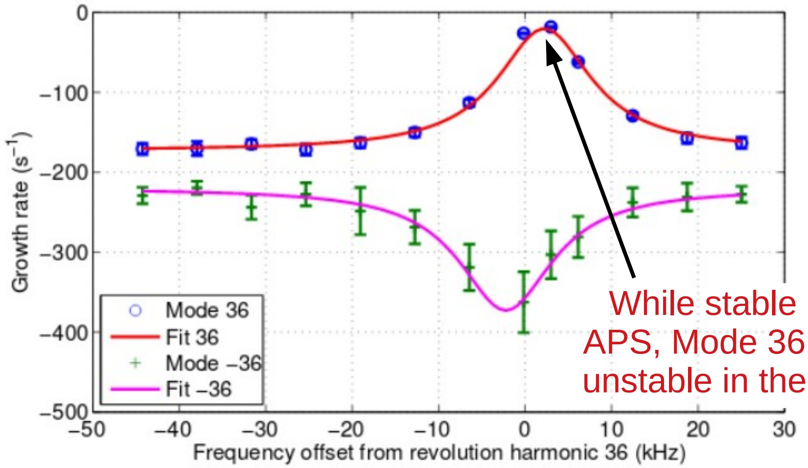
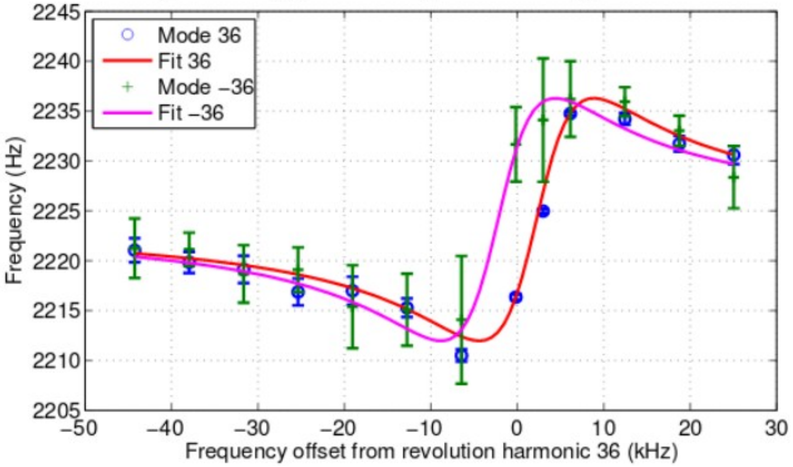
[9] M. Borland, `ELEGANT`: A flexible sdds-compliant code for accelerator simulation, Advanced Light Source Technical Report No. LS-287, 2000.

Instability can be controlled with tuning[10] and feedback

- All “dangerous” higher order modes have been identified and measured in present APS



While stable at the APS, Mode 36 can be unstable in the APS-U



[10] Final Design Report for APS-U and reports from L. Emery, S. Kallakuri

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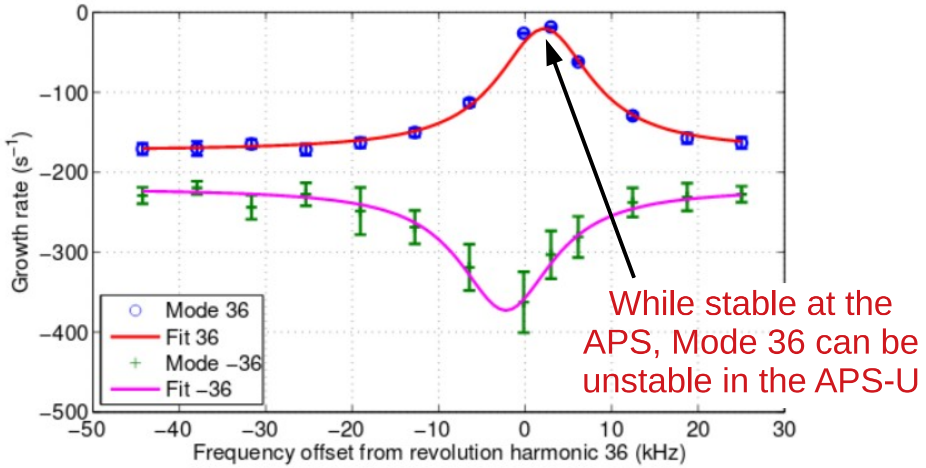

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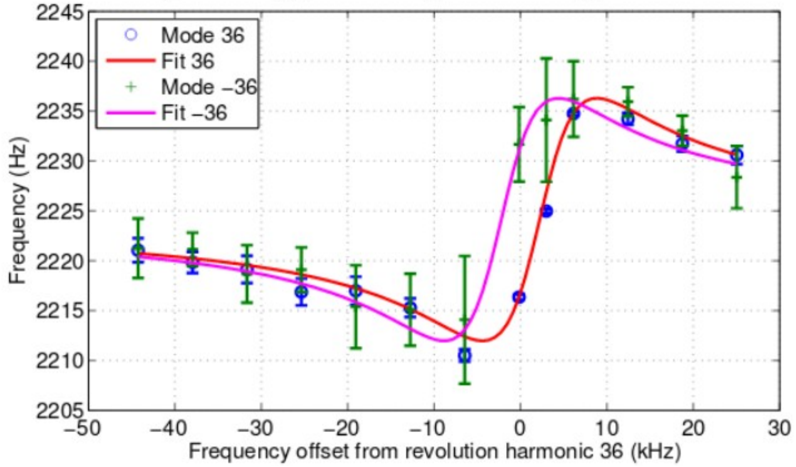

- Frequencies of modes will be shifted to “safe” regions by controlling the cavity temperature

Controlled water temperature

$$\frac{\Delta f_{\text{HOM}}}{f_{\text{HOM}}} = -\alpha_{\text{Cu}}\left(\Delta T + B\Delta P_c\right)$$

Coefficient of thermal expansion for copper, $\alpha_{\text{Cu}} \approx 10^{-5}$/°F

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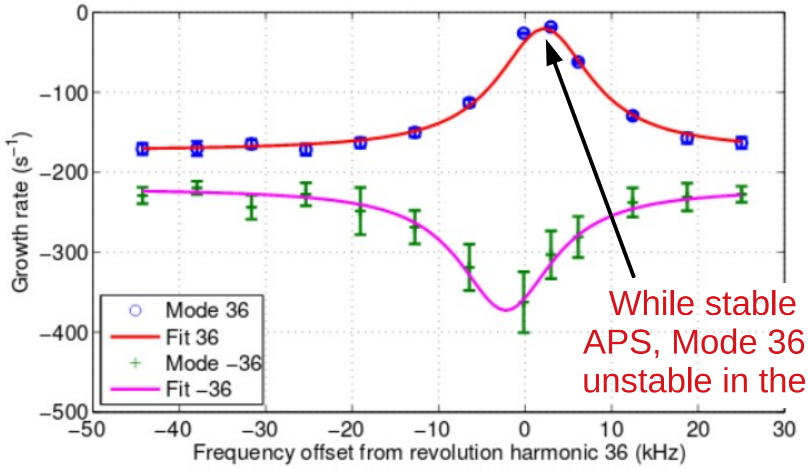

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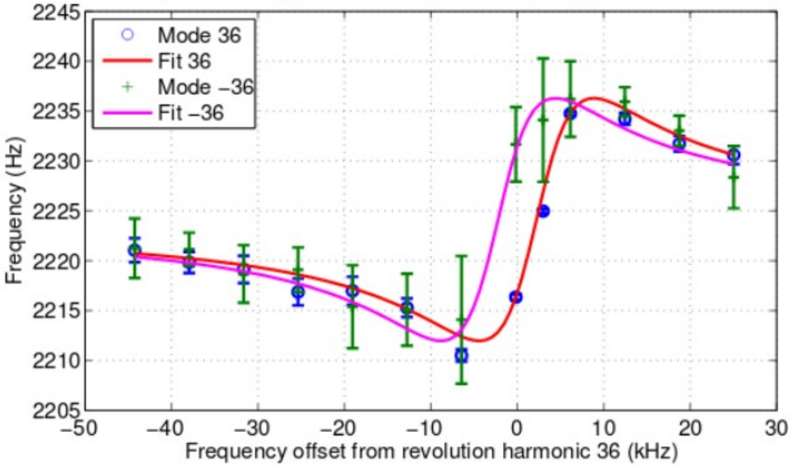

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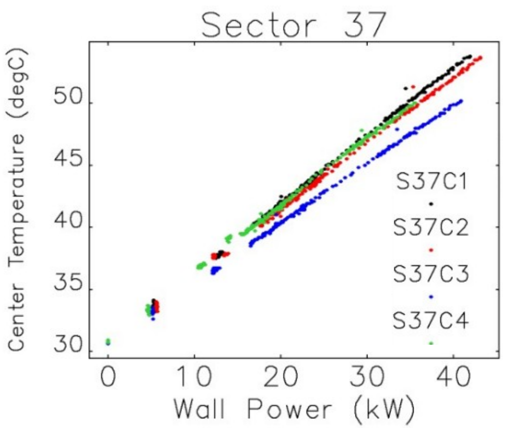

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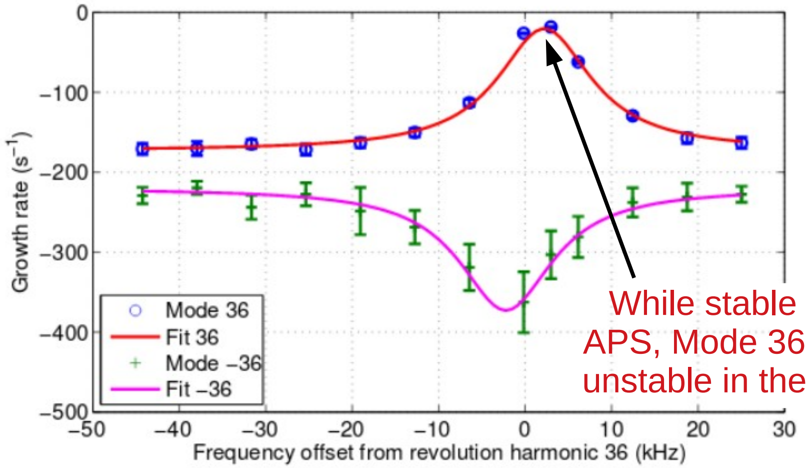

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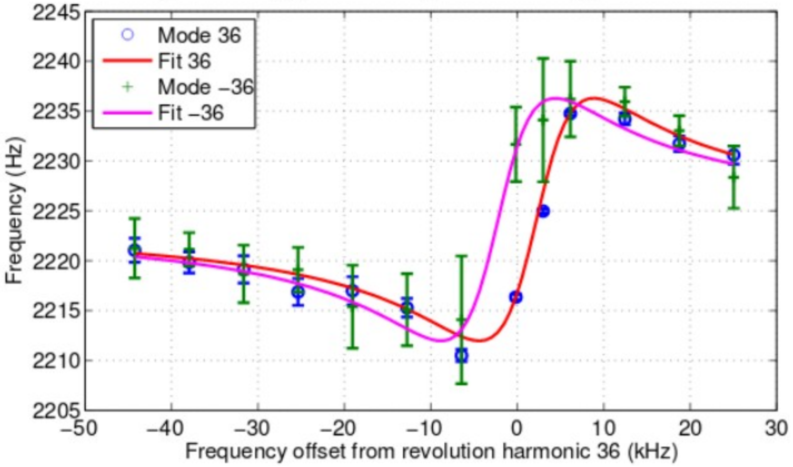

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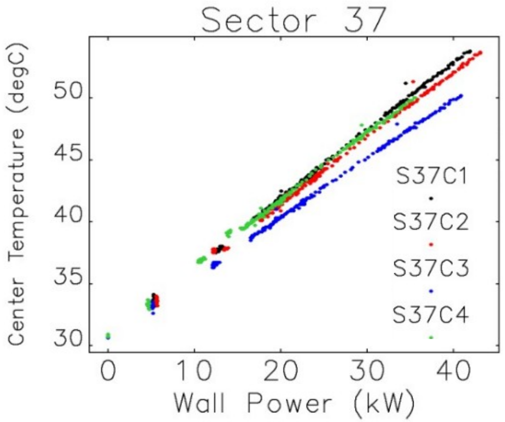
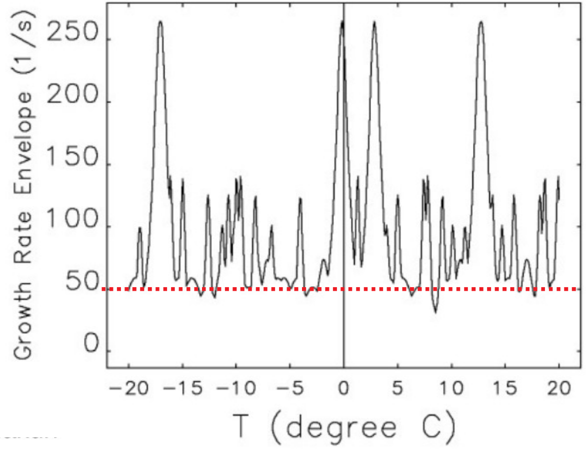


Predictions for “unfortunate” alignment of modes



Feedback damping rate is ~600 1/s

Radiation damping

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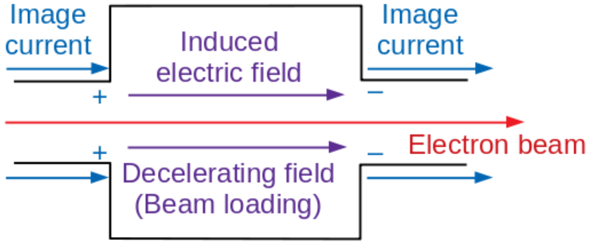
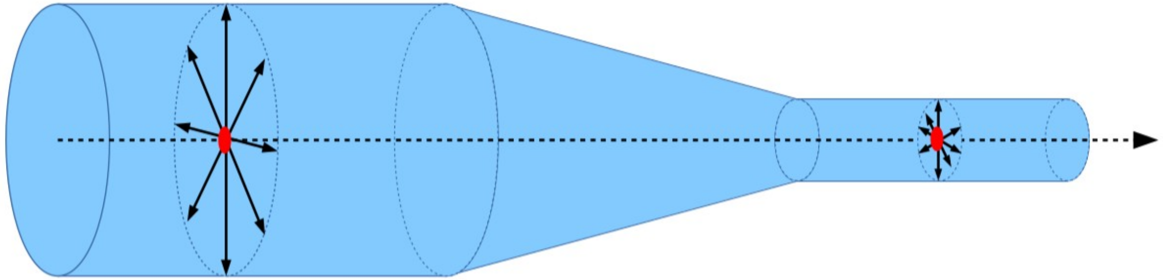
Collective forces due to impedances/wakefields[11]

- While direct space charge forces are small, particles can indirectly interact through resonant cavities
- More generally, any change in the boundary conditions will lead to collective forces



Cavity-like structures can trap electric fields from electrons



Changes in vacuum chamber cross section results in a rearrangement of fields to satisfy new boundary conditions.

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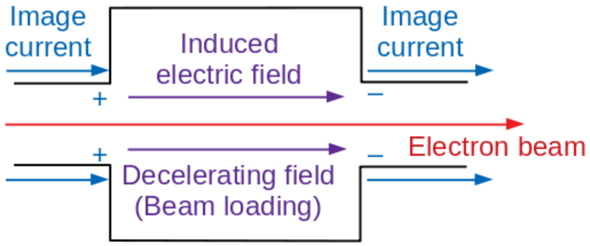

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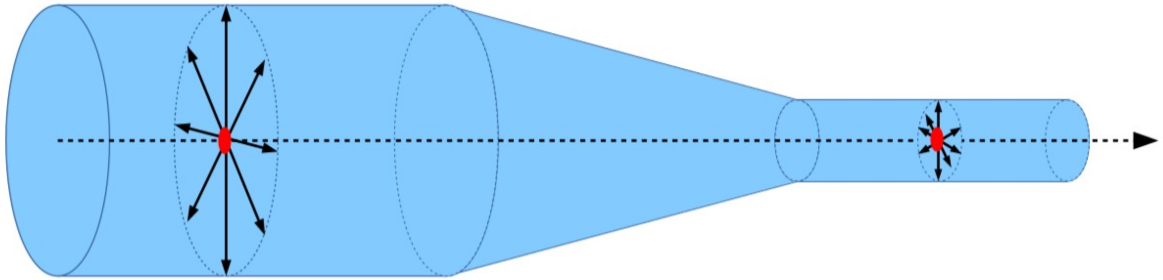
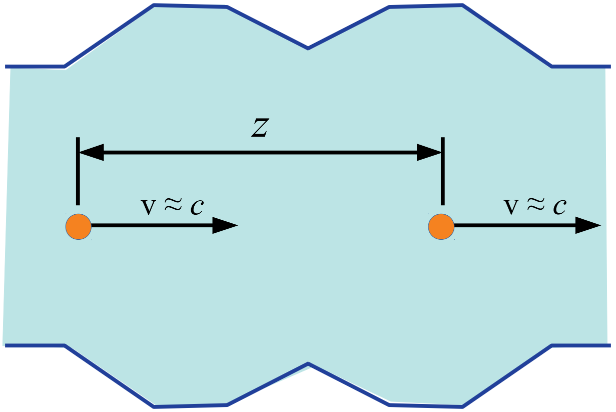


Changes in vacuum chamber cross section results in a rearrangement of fields to satisfy new boundary conditions.

- The resulting electromagnetic fields interact with particles behind the exciting charge since $\mathrm{v} \approx c$.



Longitudinal field → Energy change

$$\Delta\gamma = -\frac{e}{mc^2}\int ds\; E_z(z) \equiv -\frac{e^2}{mc^2}W_z(z)$$

Transverse fields → Angle change

$$\Delta \boldsymbol{x}'_\perp = -\frac{e}{\gamma mc^2}\int ds\;(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})_\perp$$

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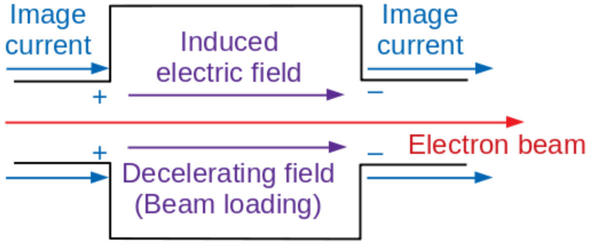

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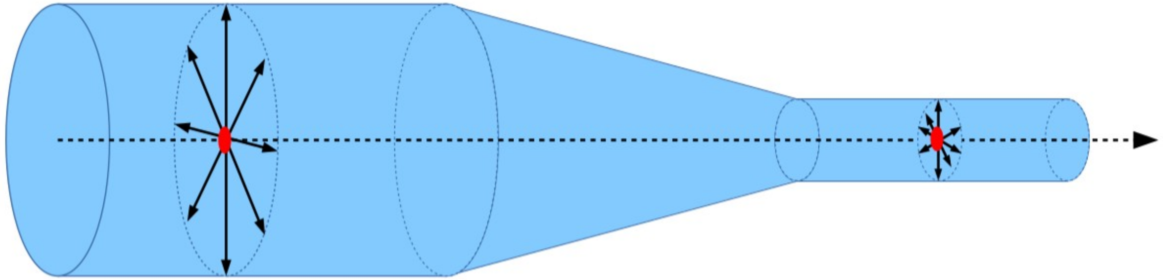

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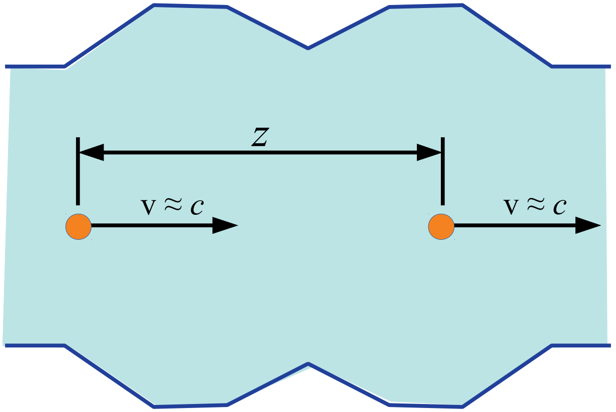

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Particles “leave behind” wakefields that quantify the impulse given to trailing particles

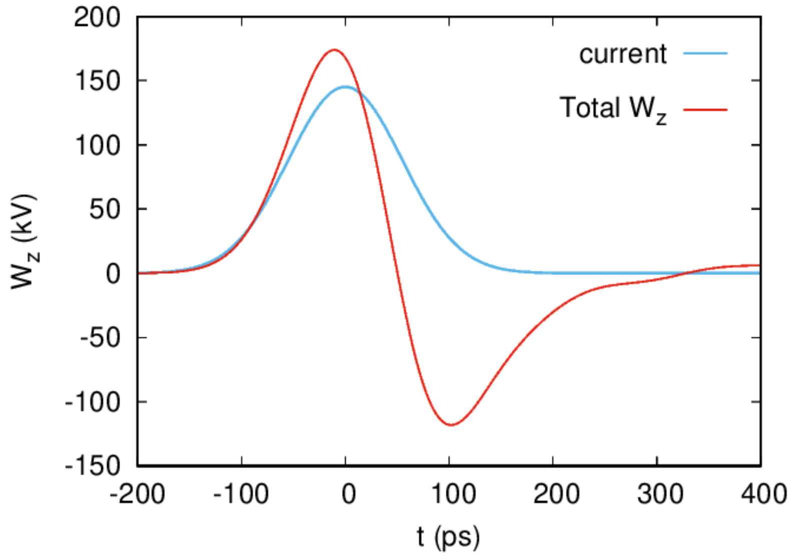
The Fourier transform of the wakefield is the impedance

$$Z(\omega) \propto \int dz\; e^{-i\omega z/c}W(z)$$

[11] B.W. Zotter and S.A. Kheifets, *Impedances and Wakes in High-Energy Particle Accelerators*, World Scientific (1998).

Longitudinal wakefields lengthen the bunch further

- Computing the wakefield associated with the entire storage ring is a long process
  - Identify all relevant contributions coming from vacuum pumps, beam position monitors, chamber transitions, cavities, etc.
  - Use simulations to calculate wakefields for each component
  - Add all contributions together
- Summing up the longitudinal wakefields from each electron yields a charge-dependent energy gain or loss at each position
  - Particles at the bunch head lose energy to those at the tail
  - Higher energy particles take a longer path around the ring → wakefields tend to stretch the bunch length even further



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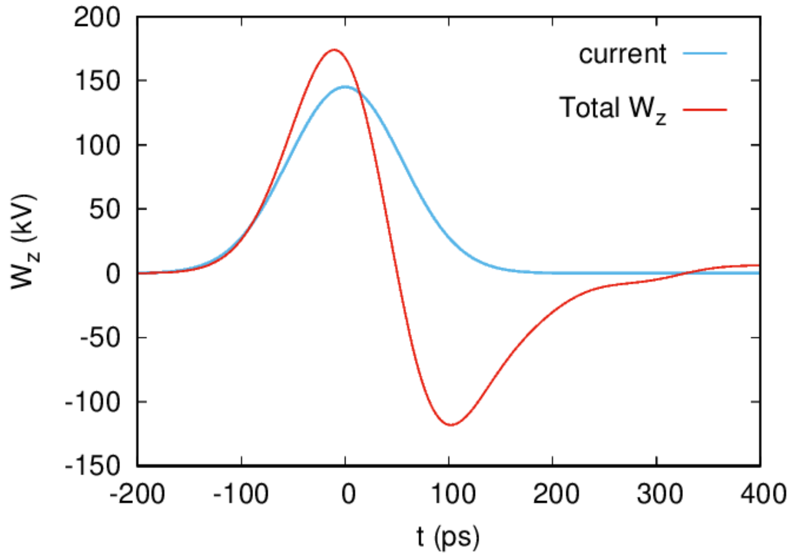
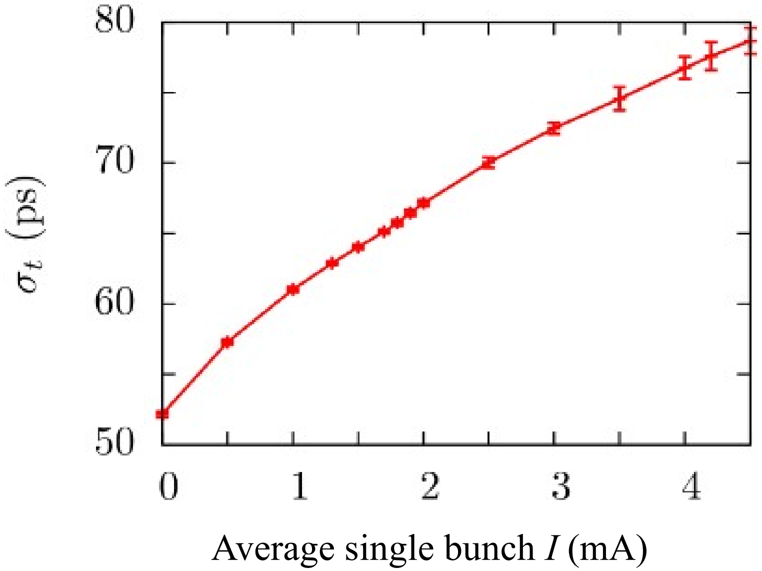
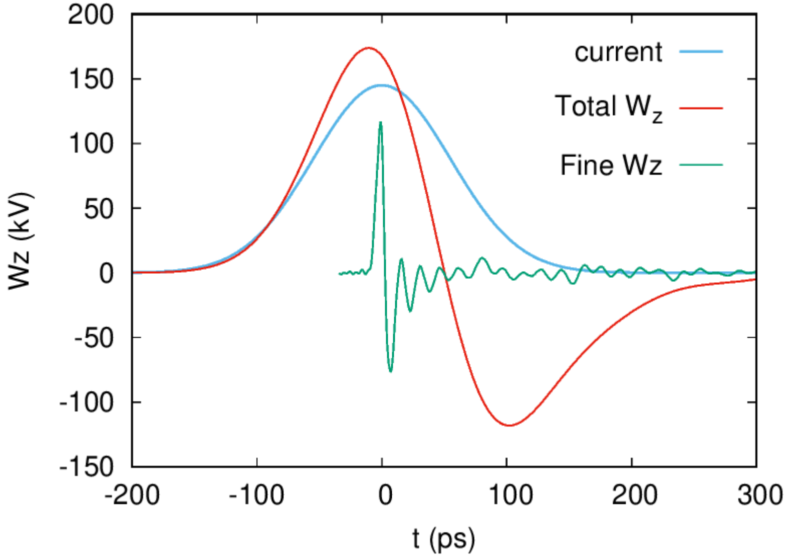
- Alternatively, one can consider the “total wakefield” as contributing an additional longitudinal potential
  - Wakefields provide additional flattening to the rf potential
  - For our applications this lengthening is benign





Wakefields can also drive single bunch instabilities

- Bunch lengthening is given by the integrated wakefield
- Fine-scale structure of the single-particle wakefields can drive a high-frequency instability that increases the energy spread



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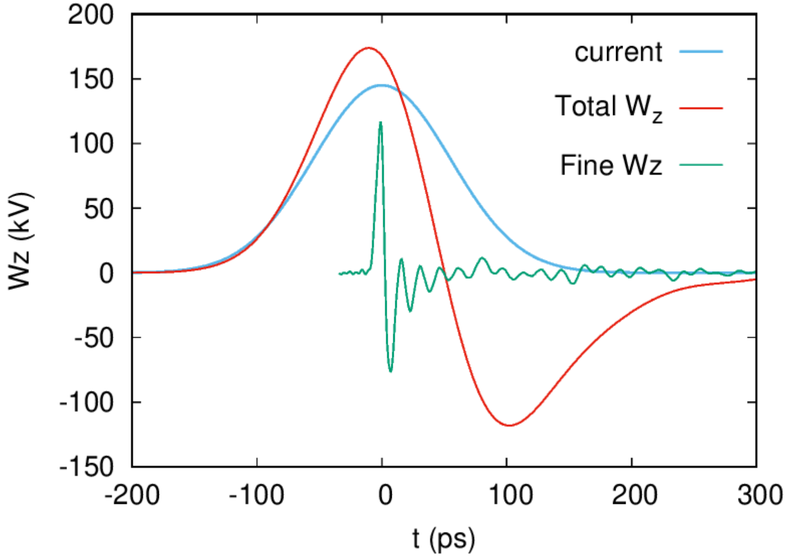

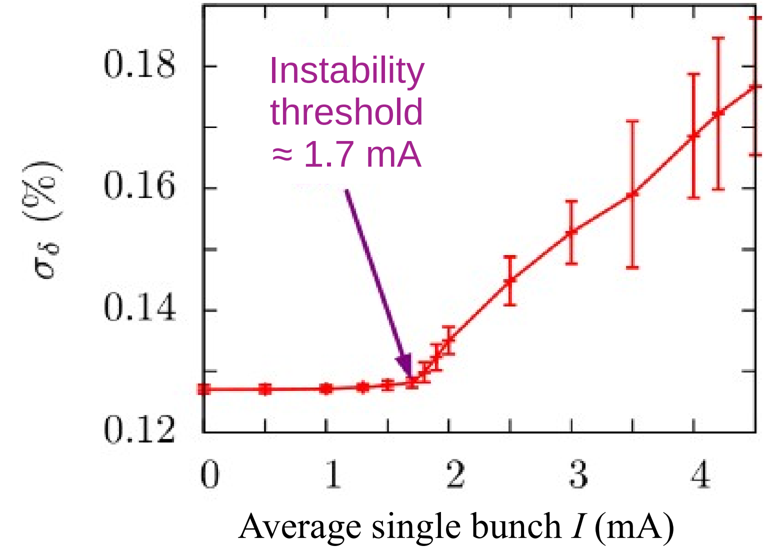

Instability threshold ≈ 1.7 mA

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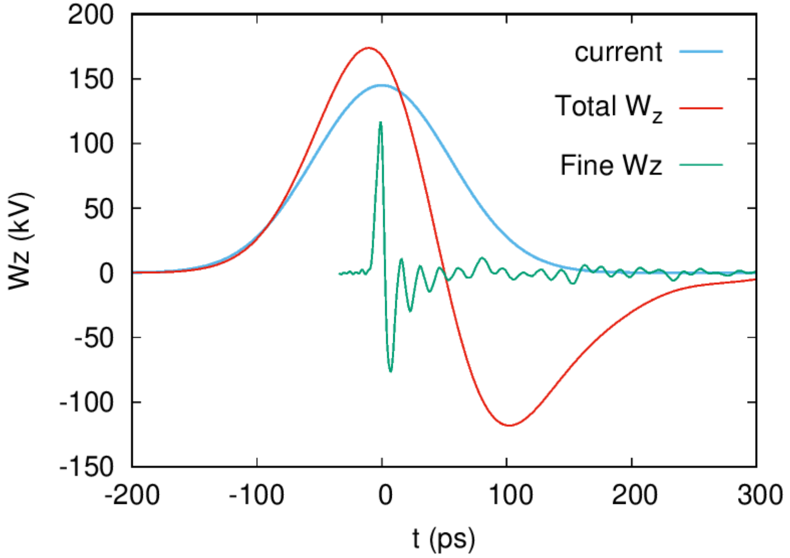
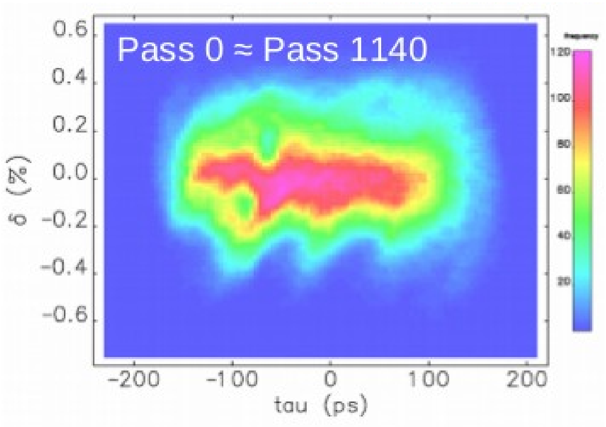
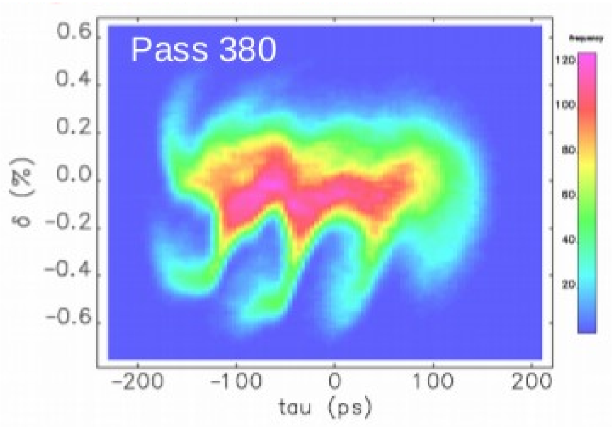
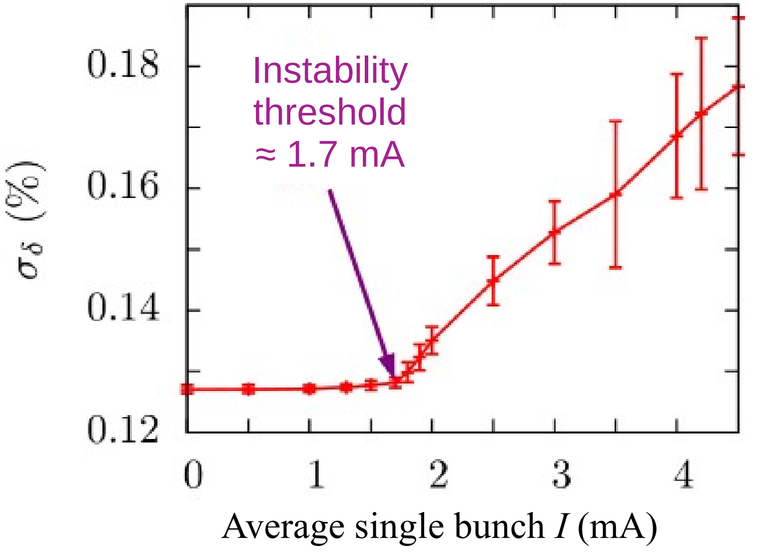
  - The longitudinal dynamics can become rather exciting at the maximum single bunch current of 4.2 mA
  - The present APS has one operating mode like this right now
  - APS-U operating with ~100 ps bunch length eliminates this instability









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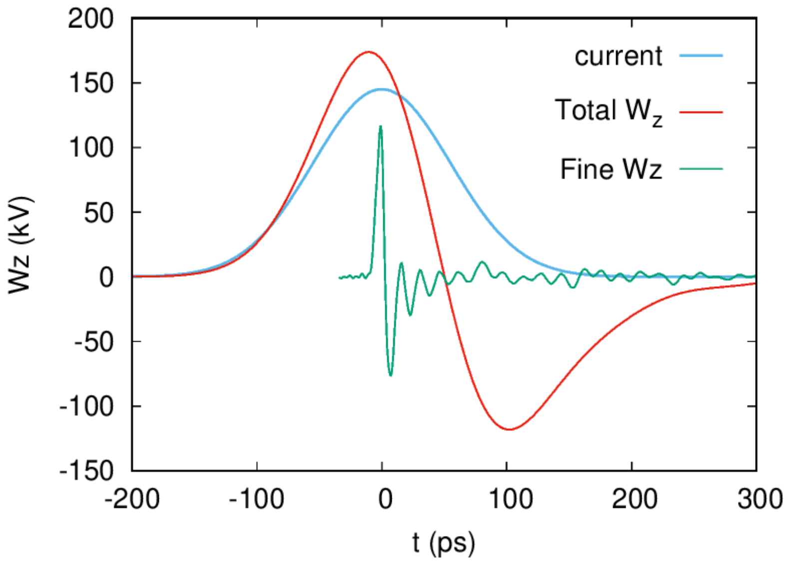
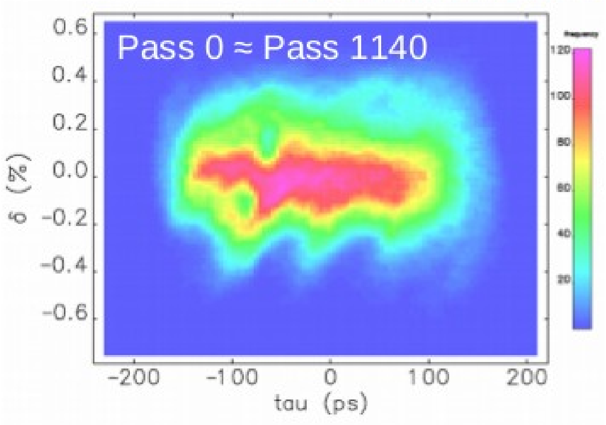
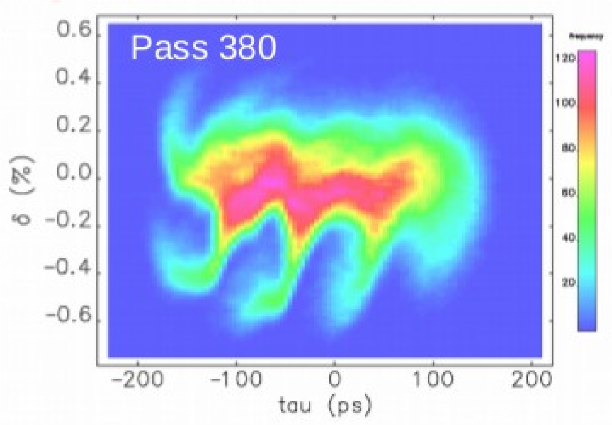
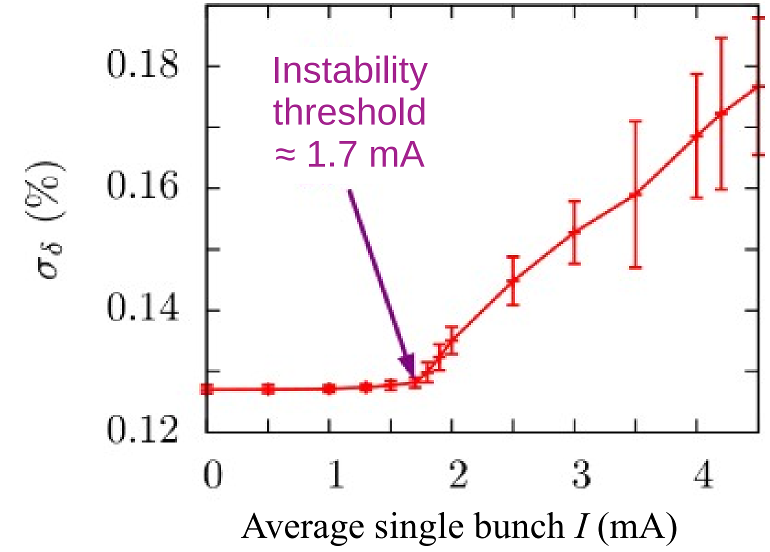
- Transverse instabilities are more concerning, since they can drive significant emittance growth and even lead to particle loss









Chromatic effects strongly influence transverse stability

- Electrons at the head of the bunch give a transverse kick to those at the tail
- Rings like the LHC at CERN control this instability with both Landau damping and feedback
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  - We would like the system to be stable, but otherwise need to know understand the feedback specs

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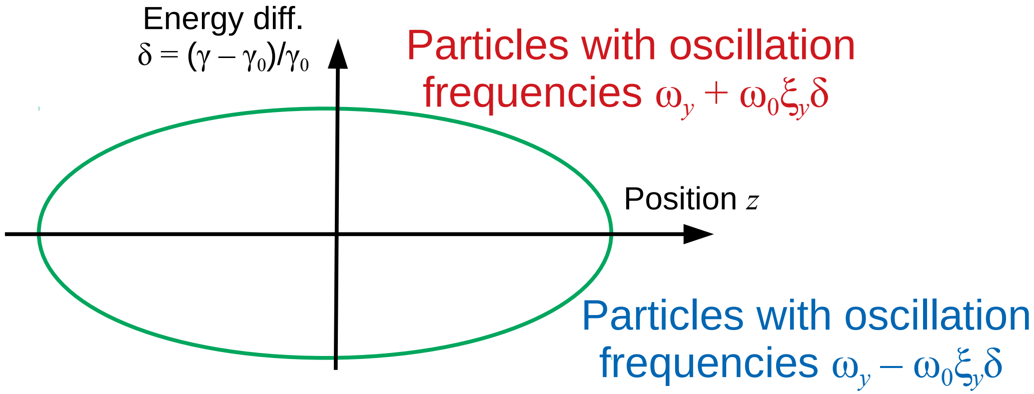

Particles with oscillation frequencies $\omega_y + \omega_0\xi_y\delta$

Particles with oscillation frequencies $\omega_y - \omega_0\xi_y\delta$

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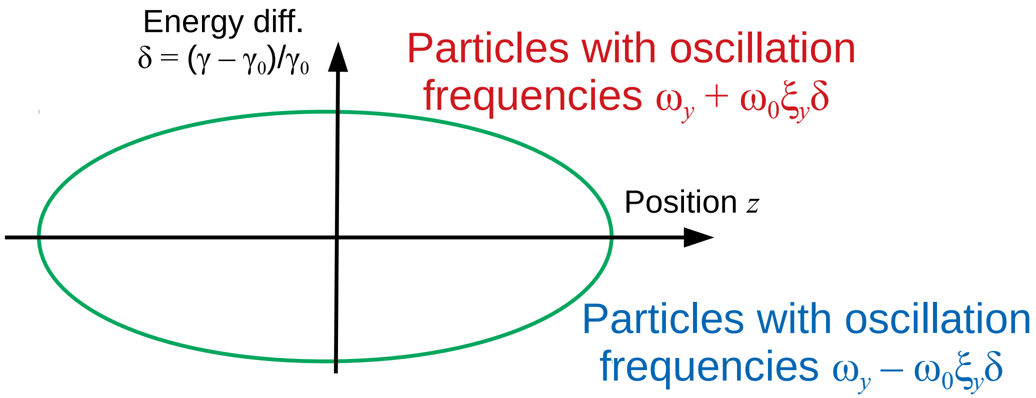
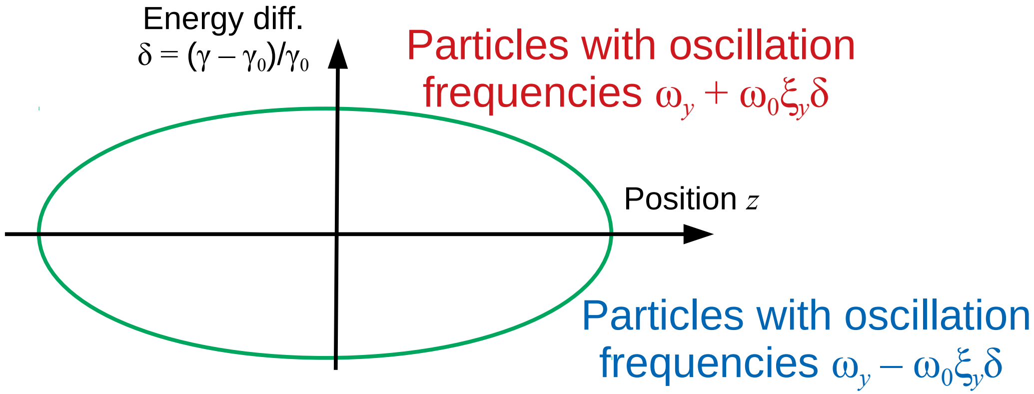


- In many other rings it is desirable to keep linear chromatic effects small, e.g., $\xi < 2$
- For the APS-U, it turns out that a large linear chromatic term is beneficial, $4 < \xi < 9$
- Furthermore, the long period of longitudinal motion leads to a phase difference across the bunch ~ $\xi_y\delta(\omega_0/\omega_z) >> 1$

Chromatic effects strongly influence transverse stability

- Electrons at the head of the bunch give a transverse kick to those at the tail
- Rings like the LHC at CERN control this instability with both Landau damping and feedback
  - For us the emittance is so small that at equilibrium the motion is very linear → no help from Landau
  - We would like the system to be stable, but otherwise need to know understand the feedback specs
- The transverse head-tail instability is also strongly affected by how the frequency depends upon energy



Particles with oscillation frequencies $\omega_y + \omega_0\xi_y\delta$

Particles with oscillation frequencies $\omega_y - \omega_0\xi_y\delta$

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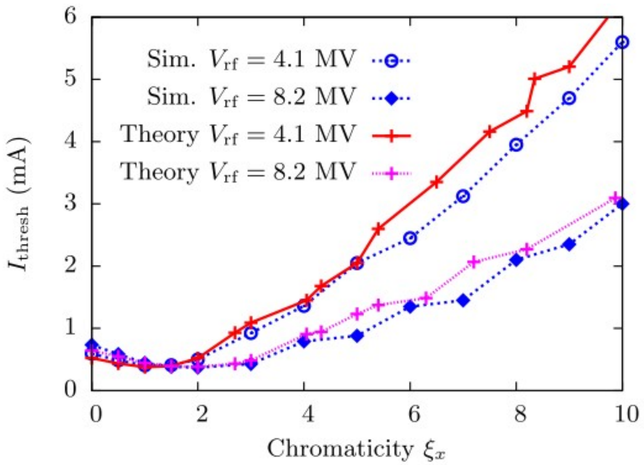
- Hence, we expect that chromatic effects will lead to significant phase mixing which will help control the instability
- Theory can be worked out for simple cases[12,13]
- Longitudinal oscillations are not linear due to bunch lengthening, so things are even more complicated for our case



[12] T. Suzuki, Fokker-Planck theory of transverse modecoupling instability, Particle Accel. **20**, 79 (1986).
[13] R.R. Lindberg, "Fokker-Planck analysis of transverse collective instabilities in electron storage rings," Phys. Rev. Accel. Beams **19** 124402 (2016)

Coupled lattice increases transverse stability for APS-U

- Including the energy dependence of the oscillation frequency, our coupled equations become[14]

Horizontal SHO: $$\frac{du_x}{dT} - i\left(\tfrac{1}{2}\{\omega_x - \omega_y\} + 2\pi\xi_x\delta\right)u_x = \frac{i\kappa}{2}u_y$$

Vertical SHO: $$\frac{du_y}{dT} + i\left(\tfrac{1}{2}\{\omega_x - \omega_y\} - 2\pi\xi_y\delta\right)u_y = \frac{i\kappa}{2}u_x$$

- The (approximately) independent degrees of freedom now involve all three planes: x, y, and z.

[14] R. Lindberg, “Collective (In)stability Near the Coupling Resonance,” Proc. IPAC, pp. 3933, THPAB075 (2021)

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- We find that the effective/shared chromatic terms can be written as the following linear combinations

$$\xi_+ = \xi_x \cos^2\theta + \xi_y \sin^2\theta$$
$$\xi_- = \xi_x \sin^2\theta + \xi_y \cos^2\theta$$

- Similarly, the effective/shared wakefields are

$$W_+^\beta(z) = \cos^2\theta\, W_x^\beta(z) + \sin^2\theta\, W_y^\beta(z)$$
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APS-U's vertical instability becomes more stable at $\theta = \pi/4$ since $\xi_x > \xi_y$.

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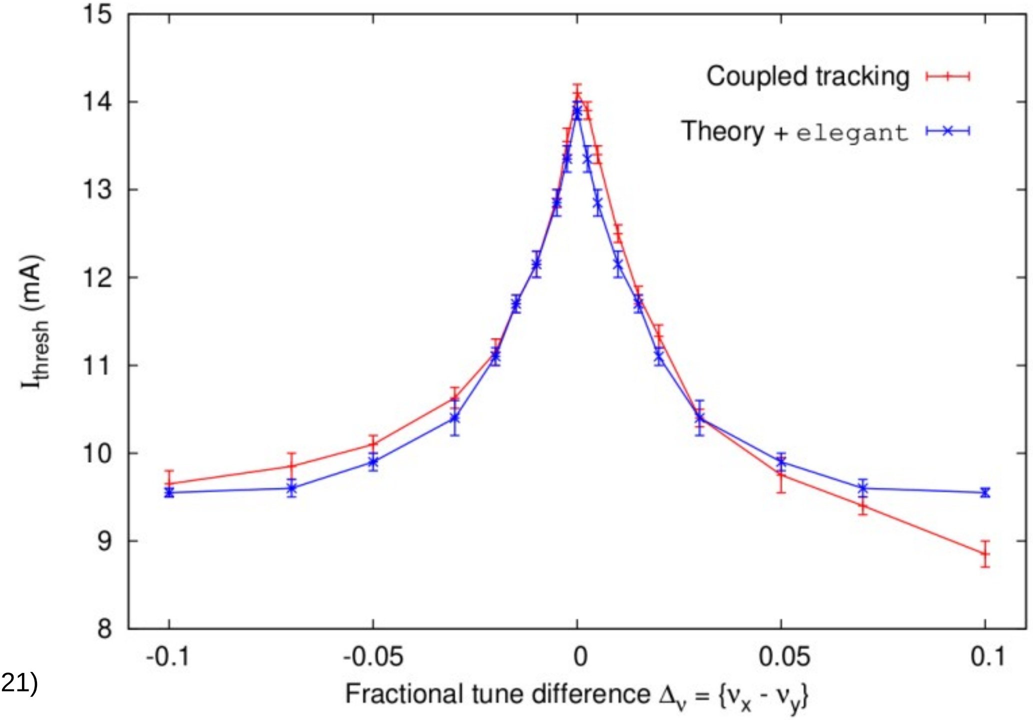

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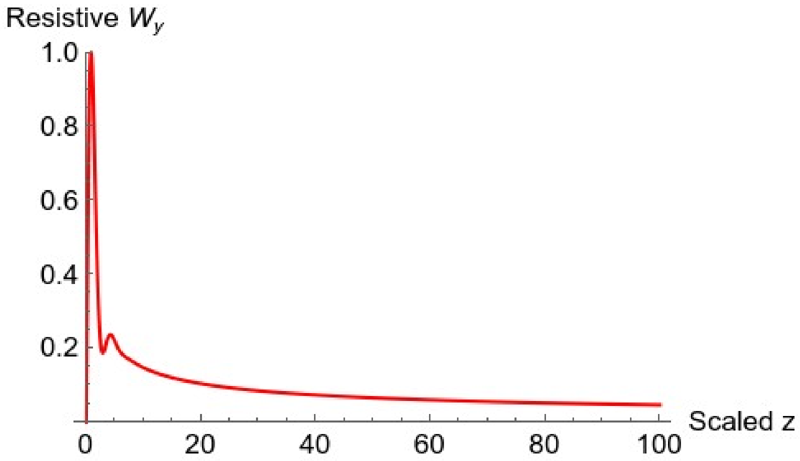
Transverse, coupled-bunch instability

- Interaction with a chamber of finite conductivity[15] leaves behind a long-range transverse wakefield $\propto z^{-1/2}$
- The transverse wakefield can excite oscillations of coupled-bunch modes



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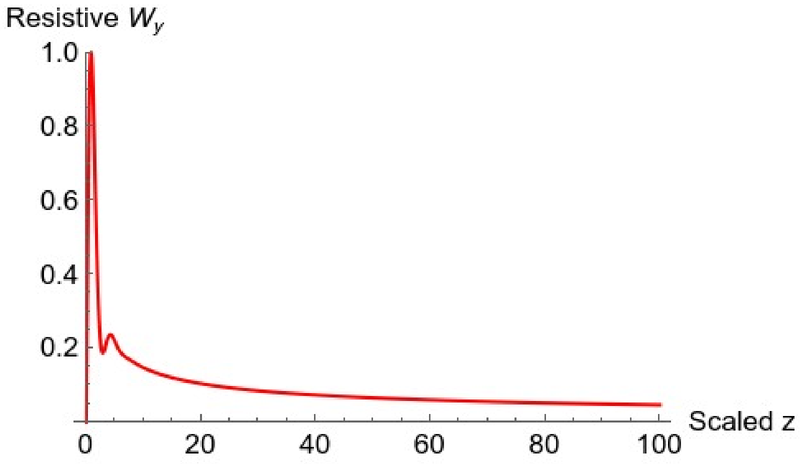

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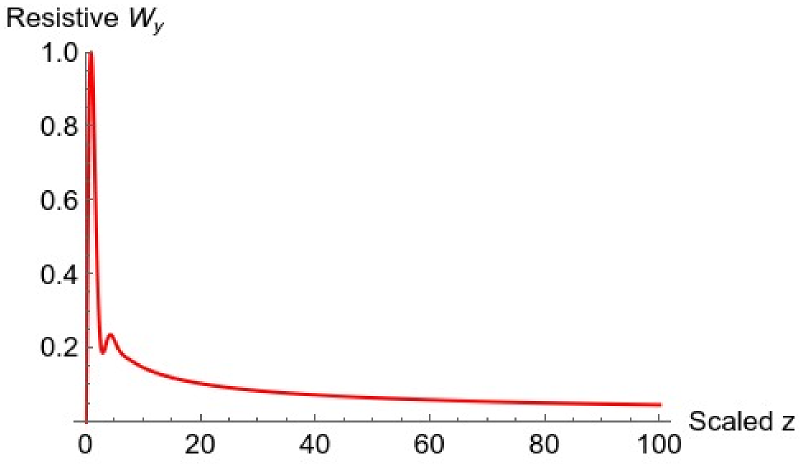
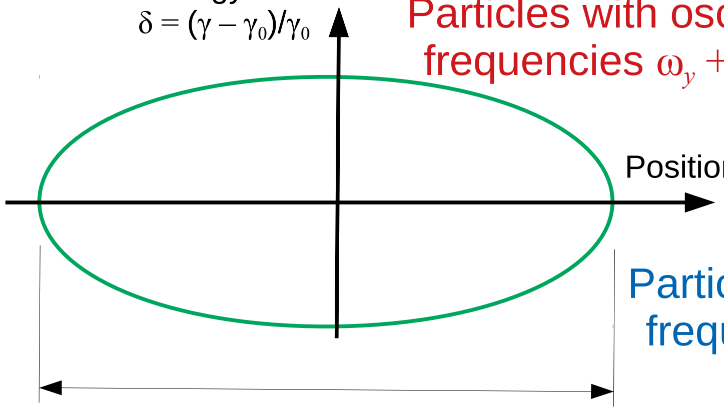
- Now, the dependence of the transverse frequency on the energy is important[16,17]
  - This leads to interesting structure in the longitudinal plane that impacts stability
  - We will characterize this effect by the characteristic transverse phase difference across the bunch length





Energy diff.
$\delta = (\gamma - \gamma_0)/\gamma_0$

Particles with oscillation frequencies $\omega_y + \omega_0\xi_y\delta$

Position z

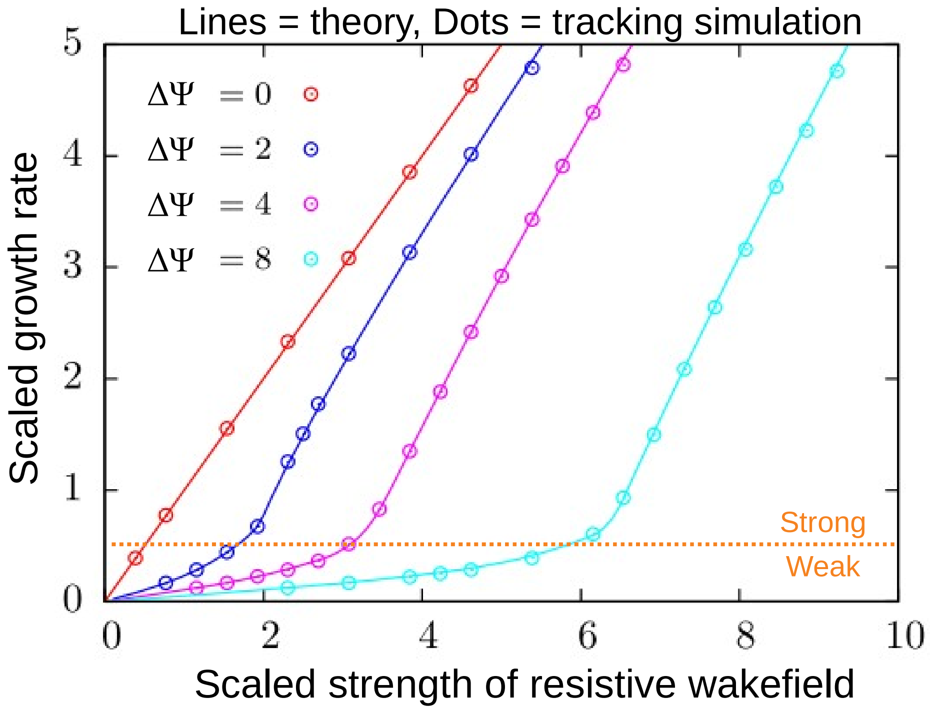
Particles with oscillation frequencies $\omega_y - \omega_0\xi_y\delta$

Phase difference $\Delta\Psi = \xi_y\delta(\omega_0/\omega_z)$

[15] O. Henry and O. Napoly, “The resistive-pipe wake potential for short bunches,” Particle Accel. **35** 235 (1991).
[16] B. Zotter and F. Sacherer, “Transverse instabilities of relativistic particle beams in accelerators and storage rings,” CERN report CERN 77-13, 175 (1977).
[17] A. Burov, “Coupled-beam and coupled-bunch instabilities,” Phys. Rev. Accel. Beams **21** 114401 (2018).

The long-range resistive wall instability is strongly damped

- Simulation and theory agree well when there is no radiation damping[18]



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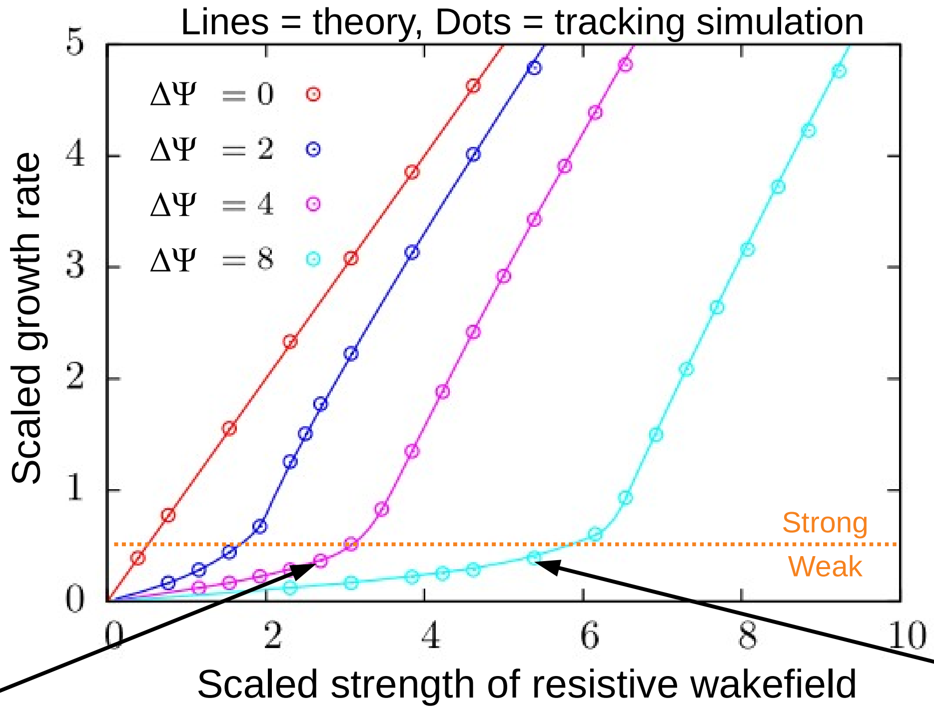

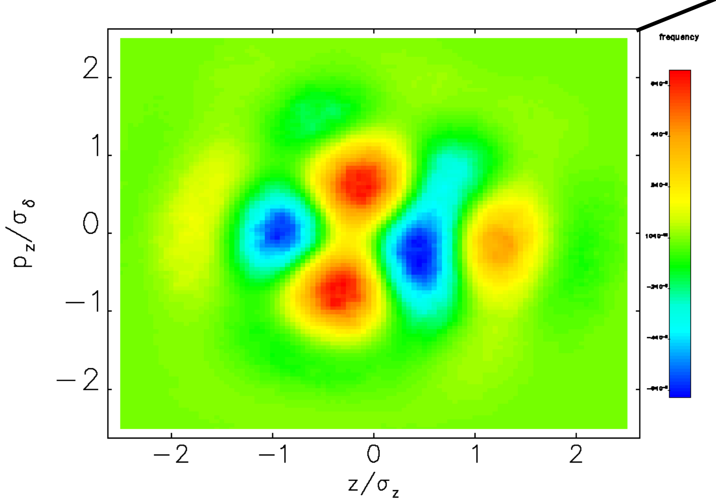

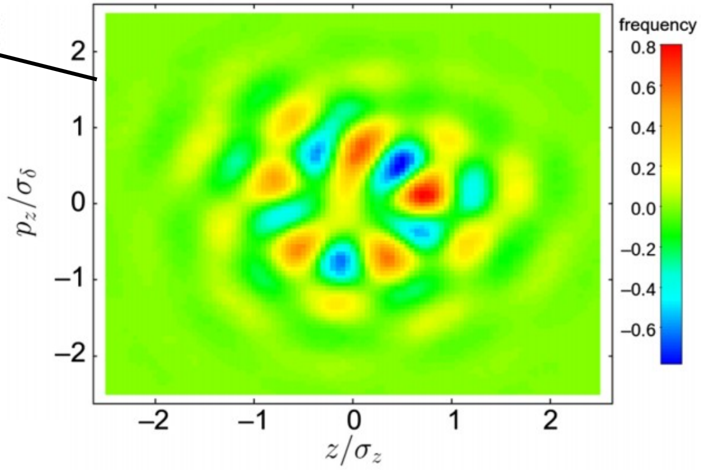

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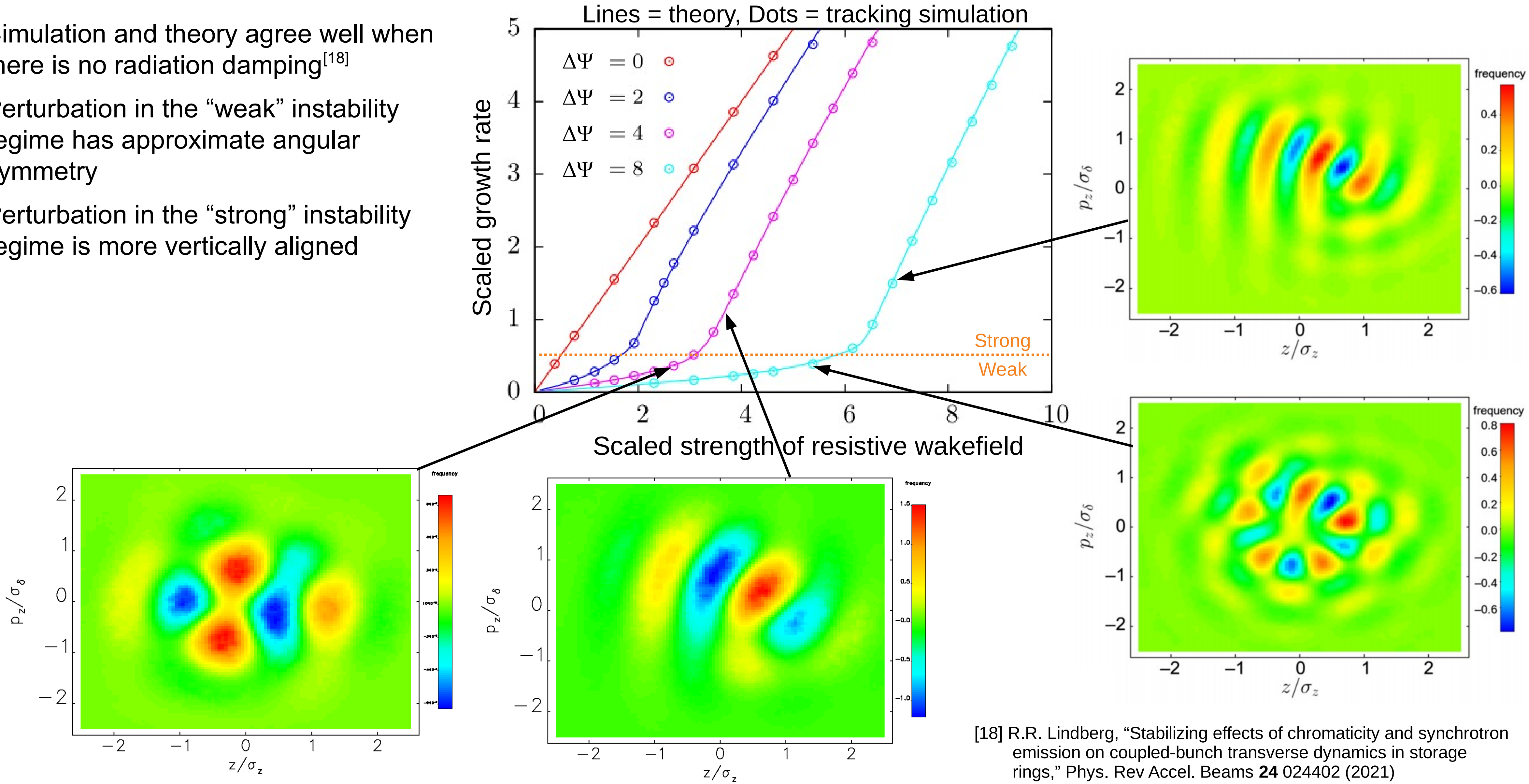

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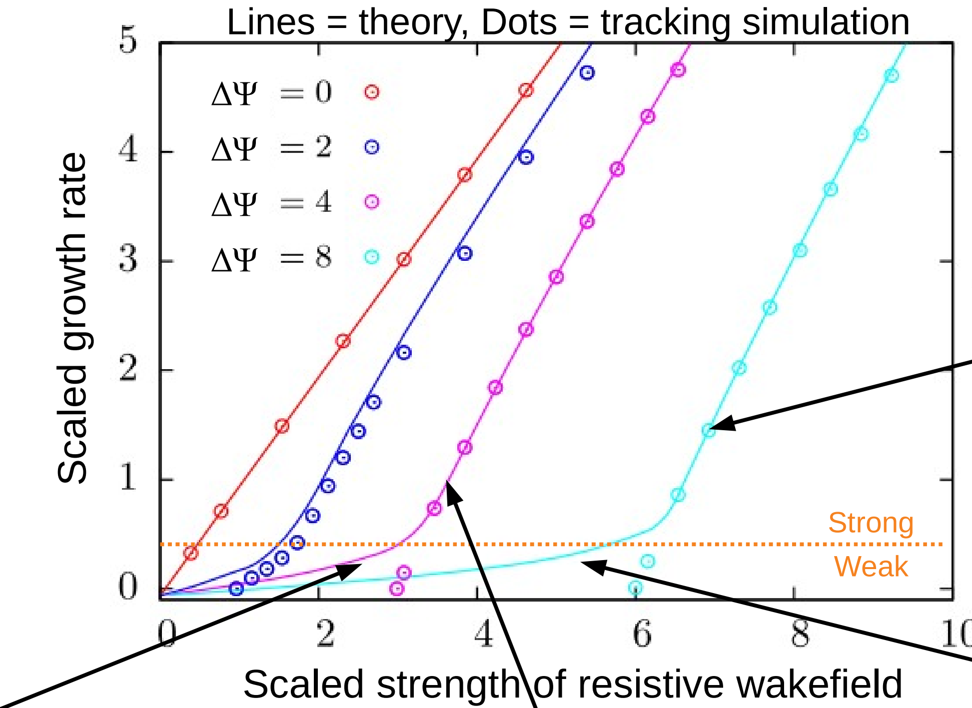
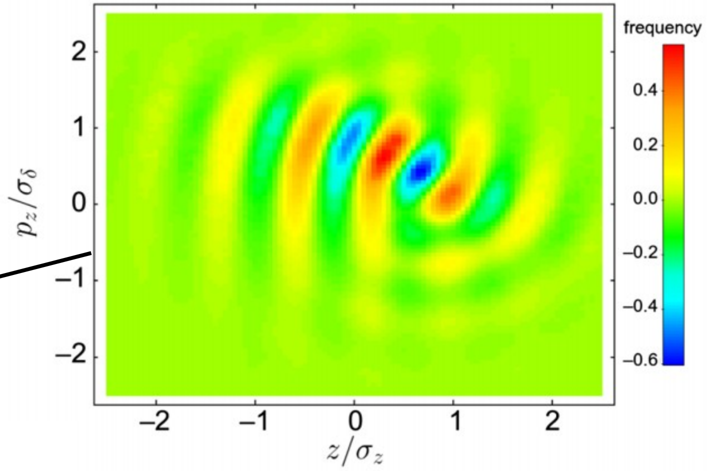
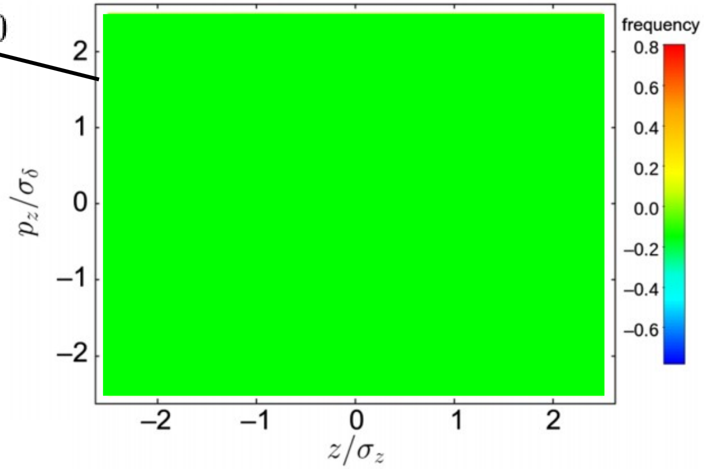
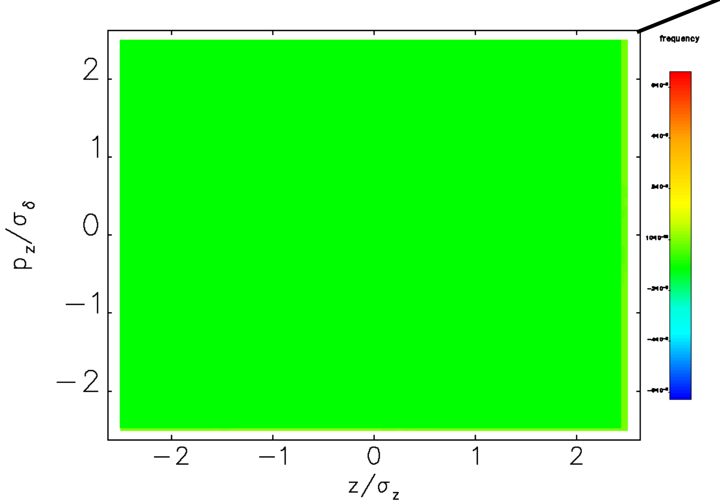
The long-range resistive wall instability is strongly damped

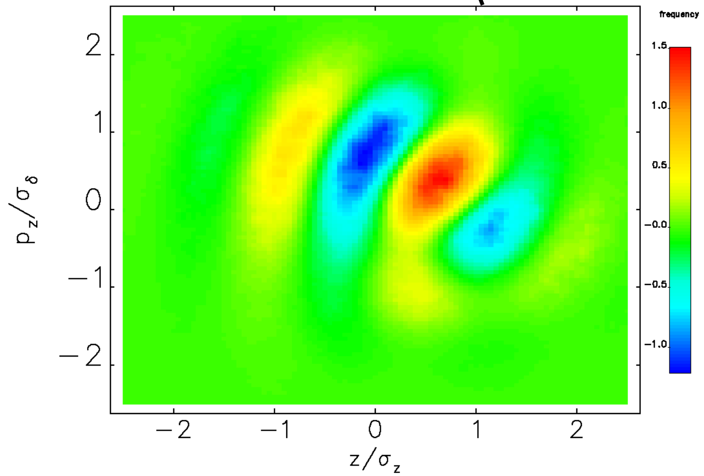
- Simulation and theory agree well when there is no radiation damping
- Perturbation in the "weak" instability regime has approximate angular symmetry
- Perturbation in the "strong" instability regime is more vertically aligned
- Adding synchrotron radiation stabilizes weak regime
- Stochastic nature of emission smooths weak instability via energy diffusion











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Conclusions

- Low emittance storage rings can provide bright X-rays for science
- Storage ring design must consider collective effects and stability
- While weak, Coulomb scattering influences storage ring design
  - Coupled focusing lattices for round beams
  - Harmonic rf cavities for bunch lengthening
- Storage ring design choices impacts collective stability
  - Long bunches from harmonic rf cavities can be longitudinally unstable
  - Coupling horizontal and vertical motion affects transverse stability
  - Energy dependence of transverse oscillations helps control instabilities
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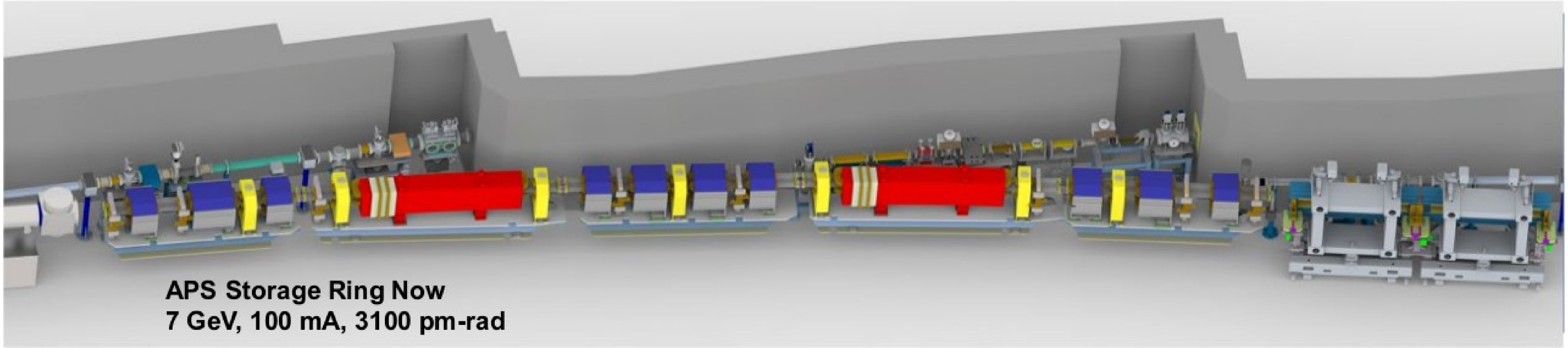
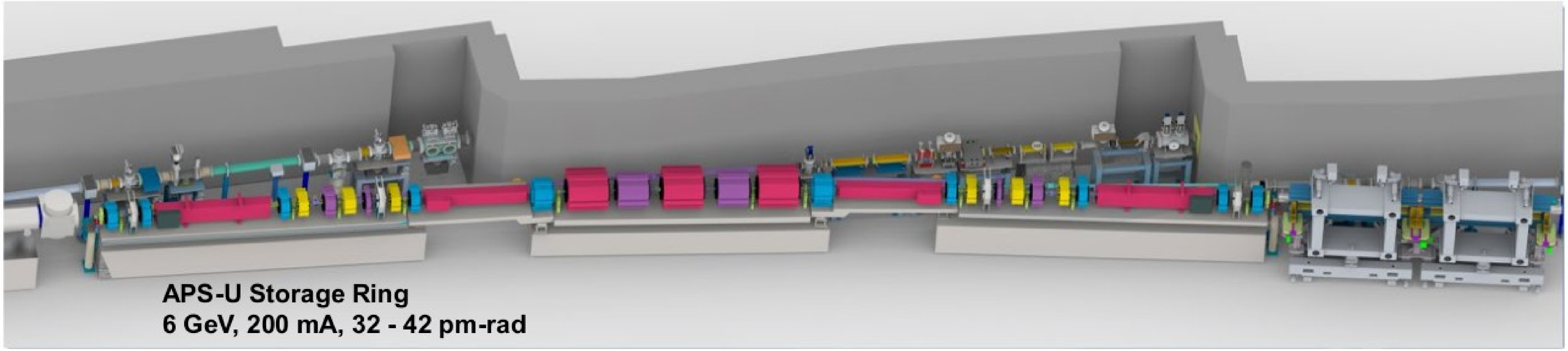
Thanks for your attention!

Bonus slides

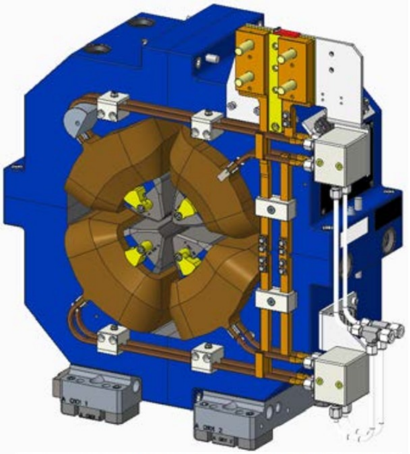
Magnet layout for APS and APS-U

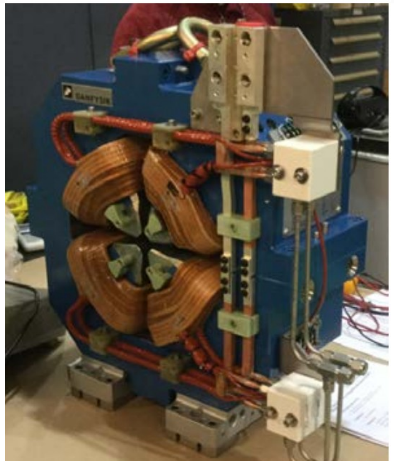




Examples of APS-U magnets

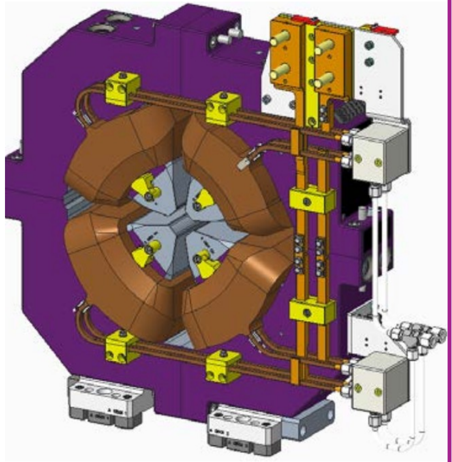
Quadrupole
CAD model





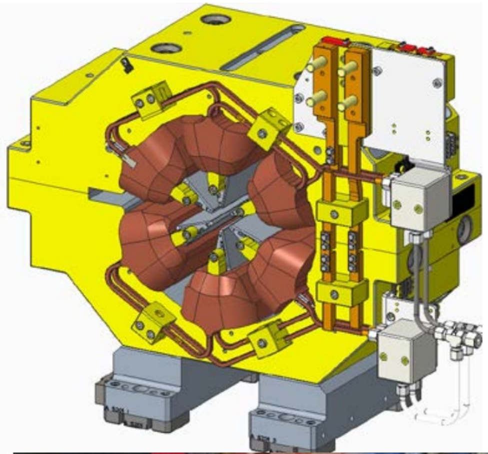
Quadrupole

Reverse bend
CAD model



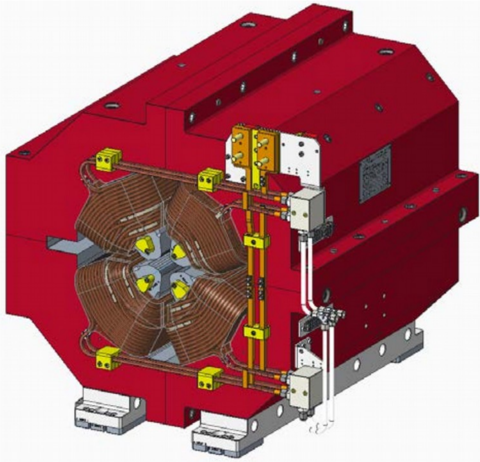
Reverse bend

Sextupole
CAD model



Sextupole

Transverse gradient
dipole CAD model



Transverse gradient
dipole

Longitudinal gradient dipole

Pictures courtesy G. Decker and M. Jaski

Energy dependence of horizontal and vertical oscillation frequencies