Analysis of CSR Microbunching

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- Derivation: KJK
- Application: ZRH

Based on ZRH & KJK

Main References

SSY (Saldin, Schneidmiller, Yurkov)

and HKS (Heifets, Krinsky, Stupakov)

Distribution function at s

$$f\left(\mathbf{X}_s ; s\right)$$

$$\mathbf{X}_s = \left\{x, x', z, \delta\right\}_s$$

In the absence of CSR, the evolution is Hamiltonian (linear):

$$\begin{aligned} d\mathbf{X}_s &= d\mathbf{X}_\tau \\ f(\mathbf{X}_s;s) &= f(\mathbf{X}_\tau;\tau) \\ &\equiv f\,(\mathbf{X}_s[\mathbf{X}_\tau];s) \\ \mathbf{X}_s[\mathbf{X}_\tau] &= R(s\rightarrow\tau)\mathbf{X}_e \end{aligned}$$

$$R\left(s \rightarrow \tau\right) = \begin{pmatrix} C(\tau \rightarrow s), & S, & 0, & \eta \\ C', & S', & 0, & \eta' \\ R_{51}, & R_{52}, & 1 & R_{56} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

With CSR

Additional energy change during $\Delta\tau$

$$\Delta\delta = \Delta\tau \frac{d\delta}{d\tau} = \Delta\tau\left(-\frac{r_e}{2\pi}\right)\int \frac{dk_1}{2\pi} Z(k_1,\tau) Nb(k_1,\tau)$$

$$e^{ik_1 z_\tau}$$

$$Z(k;\tau) = -i(1.63i - 0.94)\frac{k^{1/3}}{\rho(\tau)^{2/3}}$$

$$Nb(k,\tau) = \int d\mathbf{X}_\tau e^{-ikz_\tau} f(\mathbf{X};\tau)$$

→ Complex bunch spectrum

During $\Delta\tau$

$$f(x_s;s) = f(x_{s-\Delta\tau} - \Delta, s - \Delta\tau)$$

$$= f(x_{s-\Delta\tau}, s - \Delta\tau) - \Delta\tau\left(\frac{d\delta}{d\tau}\right)_s\left(\frac{\partial f}{\partial\delta}\right)$$

↓ Hamiltonian flow

→ keep moving back

$$f(\mathbf{X},s) = f_0(\mathbf{X},s) - \int_0^s d\tau \frac{\partial f}{\partial \delta_\tau}(\mathbf{X}_\tau;\tau)\frac{d\delta}{d\tau}$$

Hamiltonian flow from initial condition

Linearization:

$$f = \bar{f} + \hat{f}$$

perturbation

Smooth background evolving Hamiltonian

$$\bar{b} = 0$$

$$\hat{f}(\mathbf{X};s) = \hat{f}(\mathbf{X};0) - \int_0^s d\tau \frac{\partial \hat{f}}{\partial \delta}\left(\frac{d\hat{\delta}}{d\tau}\right)$$

*neglect ^ from now

$$\int d\mathbf{X}_s e^{-ik\cdot z_s} \times [\quad]$$

To find eq for bunching spectrum

$b(k;s) = b_0(k;s)$

$$-\frac{1}{N}\int d\mathbf{X}_s \int d\tau\, e^{-ikz_s} \frac{\partial \bar{f}}{\partial \delta_\tau}(\mathbf{X}_\tau;\tau)\frac{d\delta}{d\tau}$$

dX_τ

$$z_s = z_\tau + R_{51}(\tau \to s)x_\tau + R_{52}x'_\tau + \underline{R_{56}\delta_\tau}$$

δ_τ integral $\to$ integration by part

$$\frac{\partial}{\partial \delta_\tau} \to ikR_{56}(\tau \to s)$$

$$b(k;s) = b_0(k;s) + \frac{ikr_e}{\gamma}\int_0^s d\tau R_{56}(\tau \to s)\int \frac{dk_1}{2\pi} Z(k_1;\tau) b(k_1;\tau) \times \int d\mathbf{X}_0 e^{-ikz[X_0]+ik_1 z_\tau[X_0]} \bar{f}(\mathbf{X}_0)$$

HKS

chirped

$$\text{Use} \quad \bar{f}(\mathbf{X}_0) = \frac{n_0}{2\pi_0\varepsilon\sqrt{2\pi}\sigma_\delta} e^{-\frac{x_0{}^2+(\beta_0 X_0{}'+\alpha_0 X_0)^2}{2\varepsilon_x\beta_0}-\frac{(\delta_0-hz_0)^2}{2\sigma_\delta{}^2}}$$

$$d\mathbf{X}_0 = dx\, dx'\, dz\, d\delta$$

$$\text{z-integral} \to \quad \delta\left(\frac{k_1}{B(\tau)} - \frac{k}{B(s)}\right)$$

$$B(\delta) = \frac{1}{1+hR_{56}(s)} :$$ Compression factor

$k(s) = k_0 B(s)$

modulation frequency changes due to compression

- δ-integral $\rightarrow$ reduction of bunching due to energy spread smearing

$$E_\delta = e^{-k_o{}^2 U^2(s,\tau)\sigma_\delta{}^2/2}$$

$$U = B(s)R_{56}(s) - B(\tau)R_{56}(\tau)$$

- x,x′ integral $\rightarrow$ reduction due to emittance smearing

$$E_\varepsilon = e^{-\frac{k_o{}^2\varepsilon_0}{2}\beta_0\left(v-\frac{\alpha_0}{\beta_0}w\right)^2-\frac{k_0{}^2}{2\beta_0}w^2}$$

$$b(k(s);s) = b_0(k(s);s) + \int_0^s d\tau\, K(\tau,s)b(k(\tau);\tau)$$

$$K(\tau,s) = ik(s)R_{56}(\tau \rightarrow s)\frac{I(\tau)}{\gamma I_A}Z(k(\tau);\tau)E_\delta E_\delta$$

HKS

Also interesting is

Energy modulation spectrum

$$p(k,s) = \frac{1}{N}\int dX e^{-ik\cdot z}\delta f(x;s)$$

$$p(k,s) = p_0(k,s)$$

$$-\int_0^s d\tau \frac{I(\tau)}{\gamma I_A} Z(k(\tau),\tau) b(k(\tau),\tau) E_\varepsilon E_\delta$$

$$\underbrace{\left(1 - R_{56}(\tau \to s) U(\tau,s) k_0^2 B(s) \sigma_\delta^2\right)}$$

- The integral equation was solved numerically by HKS.

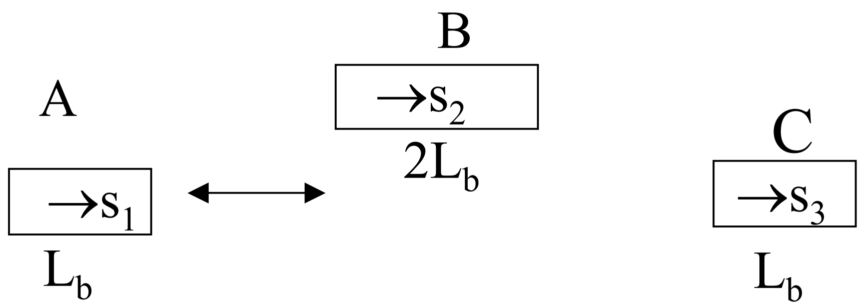
- The IE can also be solved by iteration:

$$b(k(s);s) = b_0(k(s);s) + \int_0^S d\tau\, K(\tau,\delta) b_0(k(\tau);\tau)$$

$$+ \int_0^s d\tau\, K(\tau,s) \int_0^\tau d\xi\, K(\xi,\tau) b_0(k(\xi);\xi) + \ldots$$

- Iterative solution is well-suited for studying chicane compressors.

Chicane Compressor



$$\Delta L >> L_b$$

$$R_{56}(\tau \to s) \approx 0\left(L_b{}^3/\rho_0{}^2\right): \quad \text{within the same dipole}$$

$$0\left(\Delta L L_b{}^2/\rho_o{}^2\right): \quad \text{from dipole to dipole}$$

Bunching at c:

$$b(k(s_3),s_3) = b_0(k(s_3),s_3)$$

$$+\int_A ds_1 K(s_1 s_3) b_0(k(s_1),s_1) + \int_B ds_2 K(s_2,s_3) b_0(k(s_2);s_2)$$

$$+\int_B ds_2 K(s_2,s_3) \int_A \underbrace{ds_1 K(s_1,s_2) b_0(k(s_1);s_1)}_{\text{If} >> b_0(1);\ \text{HG}}$$

Then the last term dominate → SSY.

- Compact derivation of HKS
- Energy modulation spectrum
- Iterative solution convenient for chicane
- For HG $\rightarrow$ SSY