



CSR Microbunching: Gain Calculation

Zhirong Huang, Kwang-Je Kim

Argonne National Laboratory



Integral Equation for CSR Microbunching

- For bunching parameter b at mod. wavelength $\lambda = 2\pi/k$

$$b(k; s) = b_0(k; s) + \int_0^s ds' K(s', s) b(k'; s')$$

where $b_0(k; s)$ is the bunching without CSR

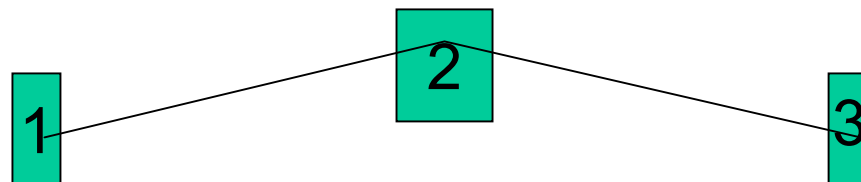
$$\text{kernel } K(s', s) = ik(s) R_{56}(s' \rightarrow s) \frac{I(s')}{\gamma \mathcal{I}_A} Z(k') \times \underbrace{\exp(\dots \varepsilon, \sigma_\delta \dots)}_{\text{Landau damping}}$$

- Given any initial condition (density and/or energy modulation), this determines the final microbunching
- Calculate $\text{gain} = b_{\text{final}}/b_{\text{initial}}$, comment on initial conditions



Staged Amplification

- Typical chicane



dipole separation $\Delta L \gg$ dipole length L_b

$$R_{56}(s' \rightarrow s) \sim \begin{cases} \frac{L_b^3}{\rho^2} & \text{within the same dipole} \\ \frac{\Delta L L_b^2}{\rho^2} & \text{from one dipole to another} \end{cases}$$

- Ignore the induced bunching from energy modulation in the same dipole (Schneidmiller et al.)
- Consider staged amplification from dipole to dipole by setting $K(s', s) = O(L_b/\Delta L) = 0$ if $s - s' < \Delta L$



Iterative Solution

- Integral equation can be solved by two iterations

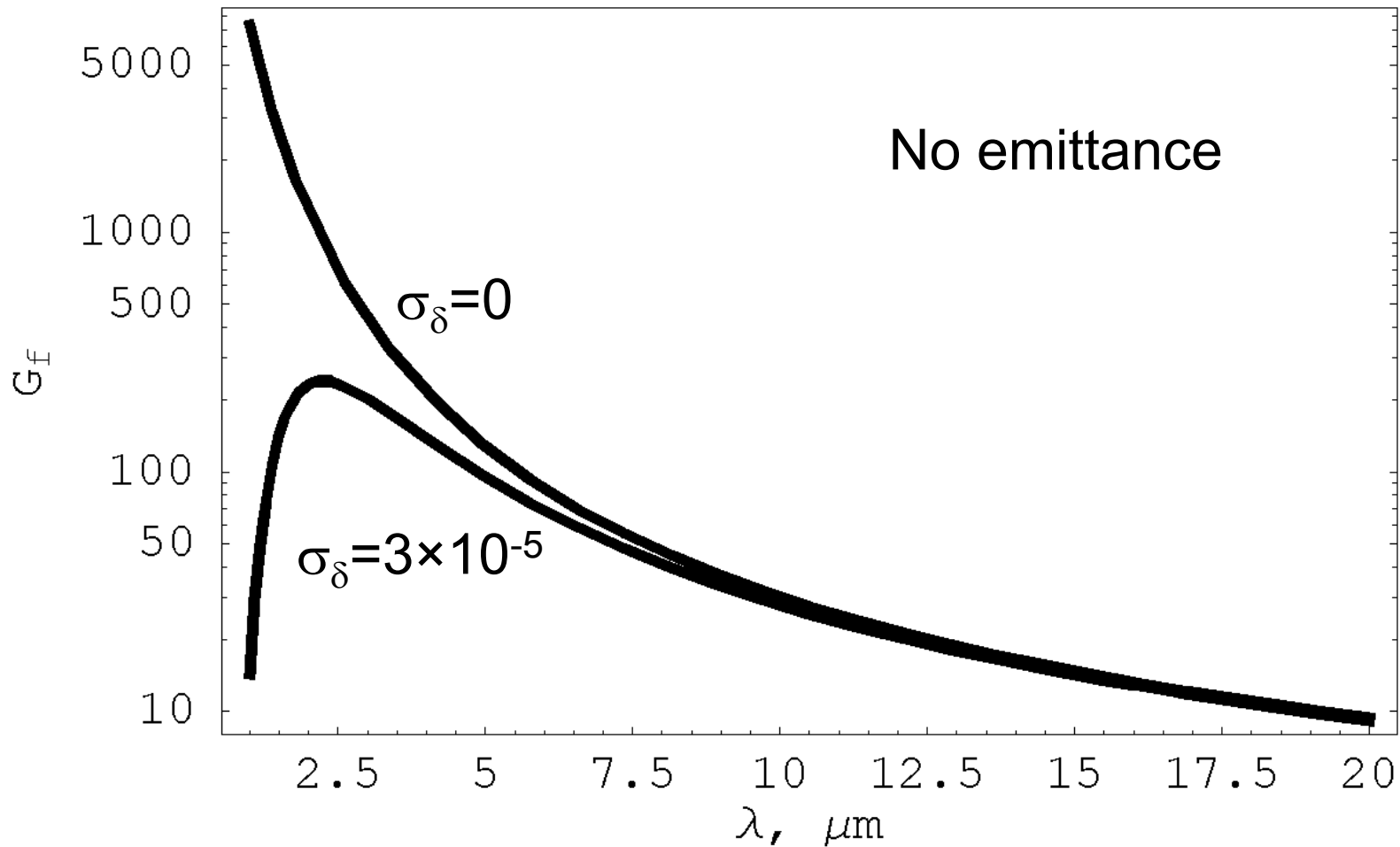
$$b(k; s) = b_0(k; s) + \underbrace{\int_0^s ds' K(s', s) b_0(k'; s')}_{\text{one - stage amplification}}$$

$$+ \underbrace{\int_0^s ds' K(s', s) \int_0^{s'} ds'' K(s'', s') b_0(k''; s'')}_{\text{two - stage amplification (no compression and no emittance} \rightarrow \text{Schneidmiller et al.)}}$$

two - stage amplification (no compression and no emittance \rightarrow Schneidmiller et al.)

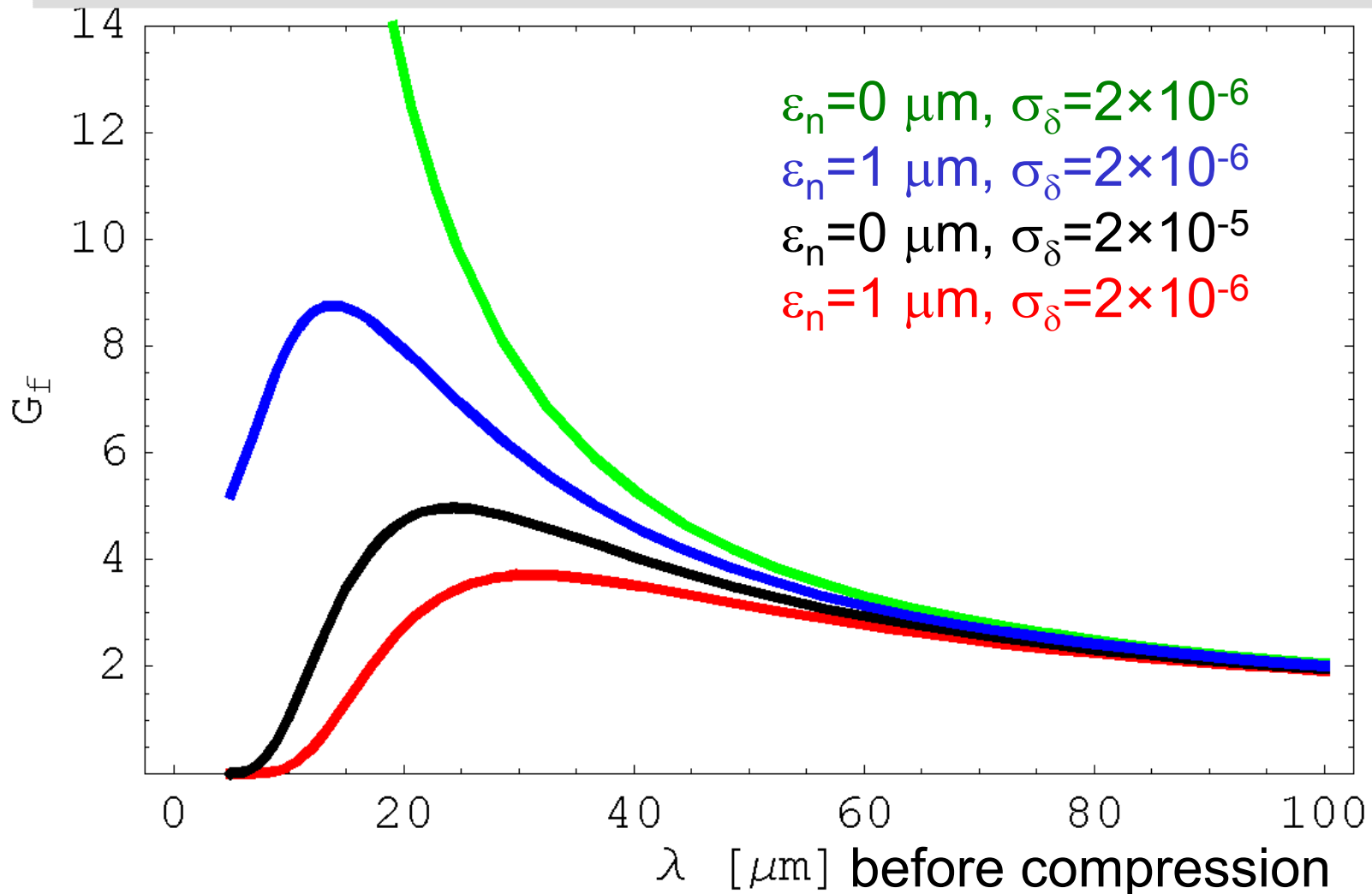
$$= b_0(k; s) + \underbrace{I_f(1 \rightarrow 3) + I_f(2 \rightarrow 3)}_{\text{dominant in low-gain}} + \underbrace{I_f^2(1 \rightarrow 2 \rightarrow 3)}_{\text{high-gain}}$$

Berlin CSR Benchmark: No Compression

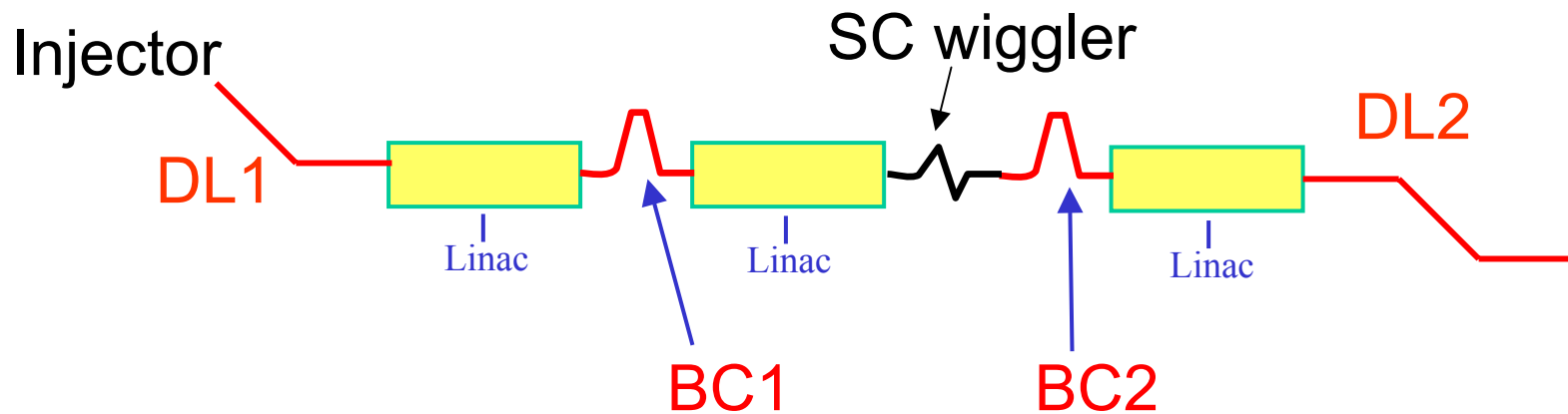




Berlin CSR Benchmark: Compression



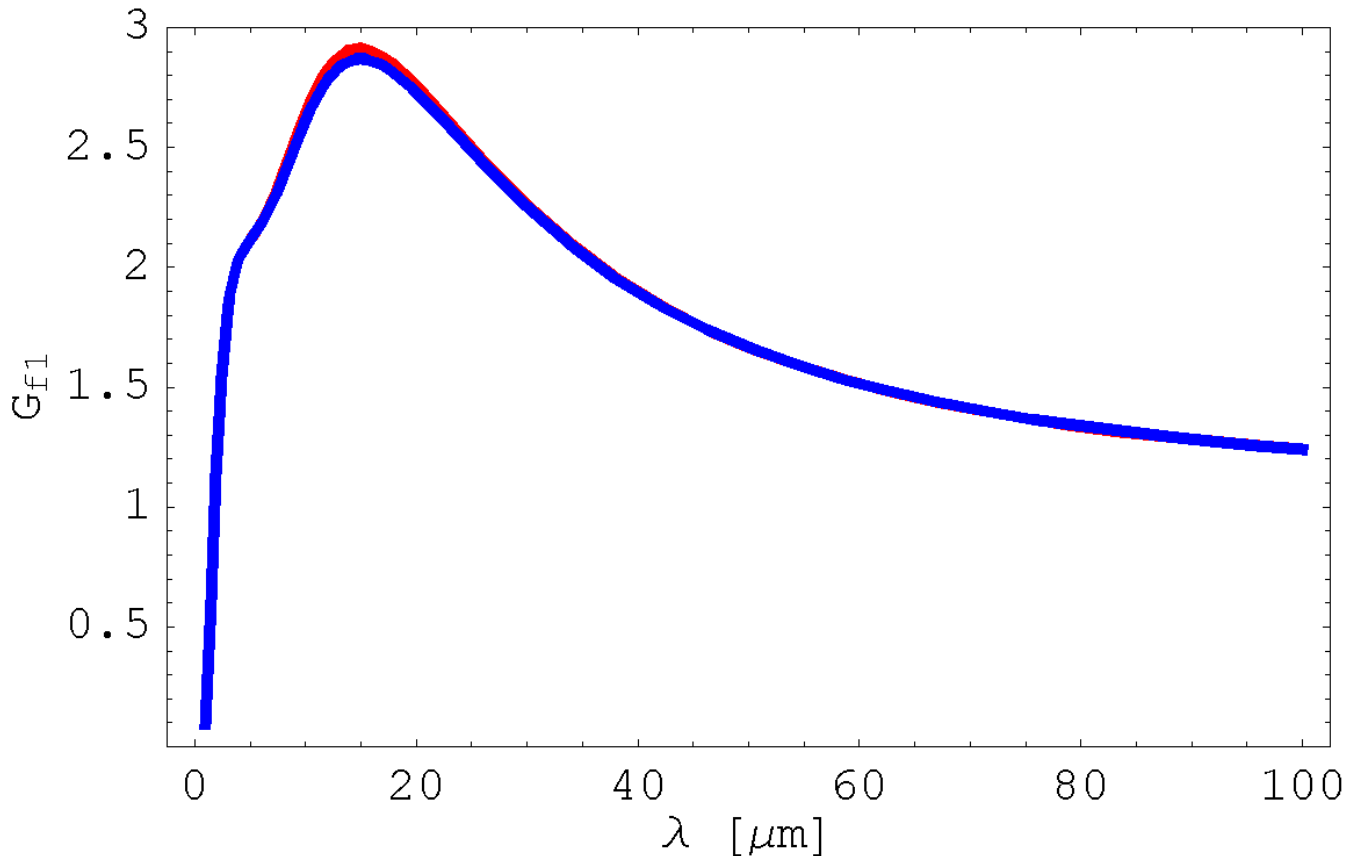
LCLS Bunch Compressors (BC)



| | | |
|--------------------|----------------------|---|
| Energy | 250 MeV | 4.54 GeV |
| Compressor current | 480 A | 4000 A |
| Norm. emittance | 1 mm | 1 mm |
| Energy spread | 1.2×10^{-5} | 3×10^{-6} (3×10^{-5}) |



BC1 Gain of Density Modulation



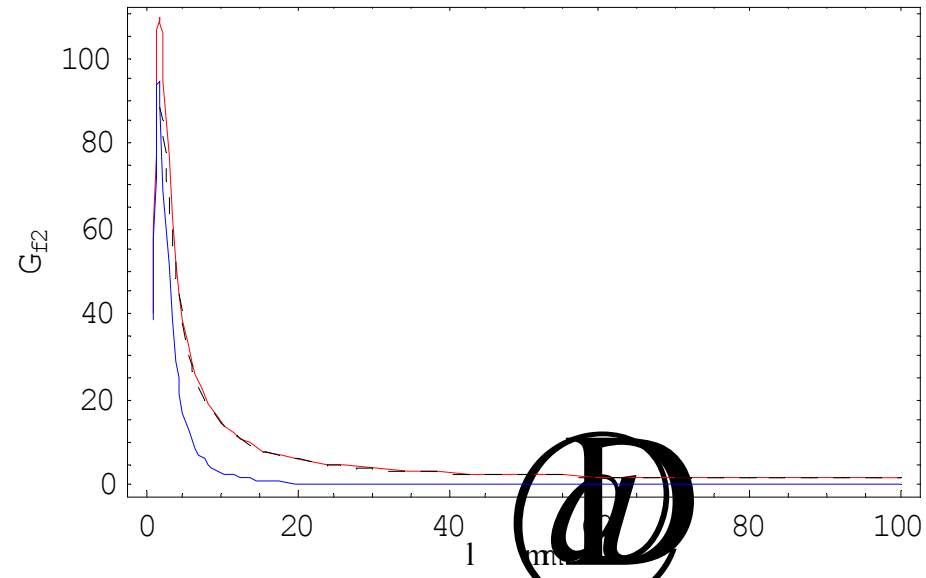
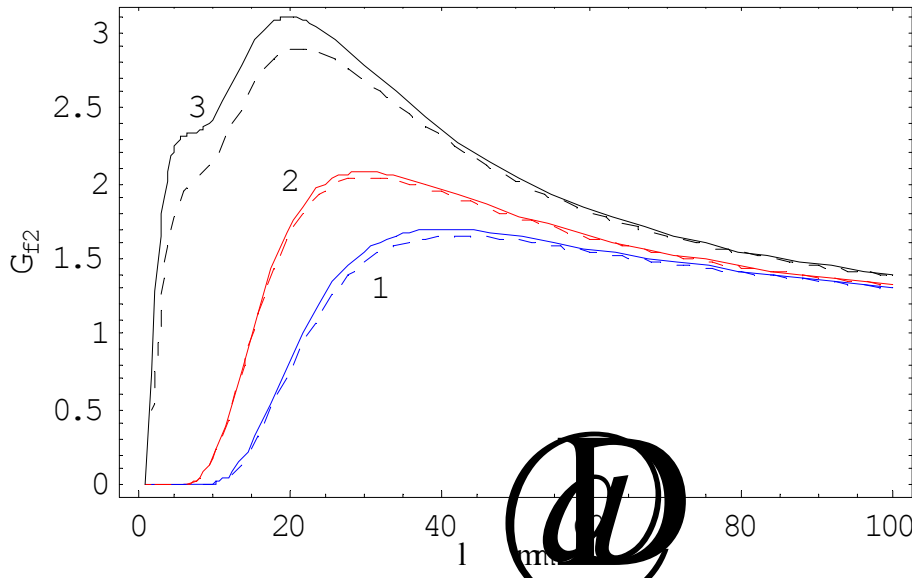
$$\sigma_{\delta} = 1.2 \times 10^{-5},$$
$$\varepsilon_n = 1 \mu\text{m}$$

- 1) 1-stage
- 2) 1+2 stage

- At low gain, one-stage amplification dominates



BC2 Gain of Density Modulation



- 1) $\sigma_\delta = 3 \times 10^{-5}$, $\varepsilon_n = 1 \mu\text{m}$
- 2) $\sigma_\delta = 3 \times 10^{-5}$, $\varepsilon_n = 0 \mu\text{m}$
- 3) $\sigma_\delta = 3 \times 10^{-6}$, $\varepsilon_n = 1 \mu\text{m}$

$\sigma_\delta = 3 \times 10^{-6}$, $\varepsilon_n = 0 \mu\text{m}$
Two-stage amplification
dominates at high-gain

- Analytical solutions agree with the numerical solutions (from Heifets/Stupakov/Krinsky, dashed curves)



Energy Modulation

- CSR Induced energy modulation in bends

$$\Delta p(s) \approx -\int_0^s ds \frac{I(s')}{\gamma I_A} Z(s') b(s') \exp(\dots \varepsilon, \sigma_\delta \dots)$$

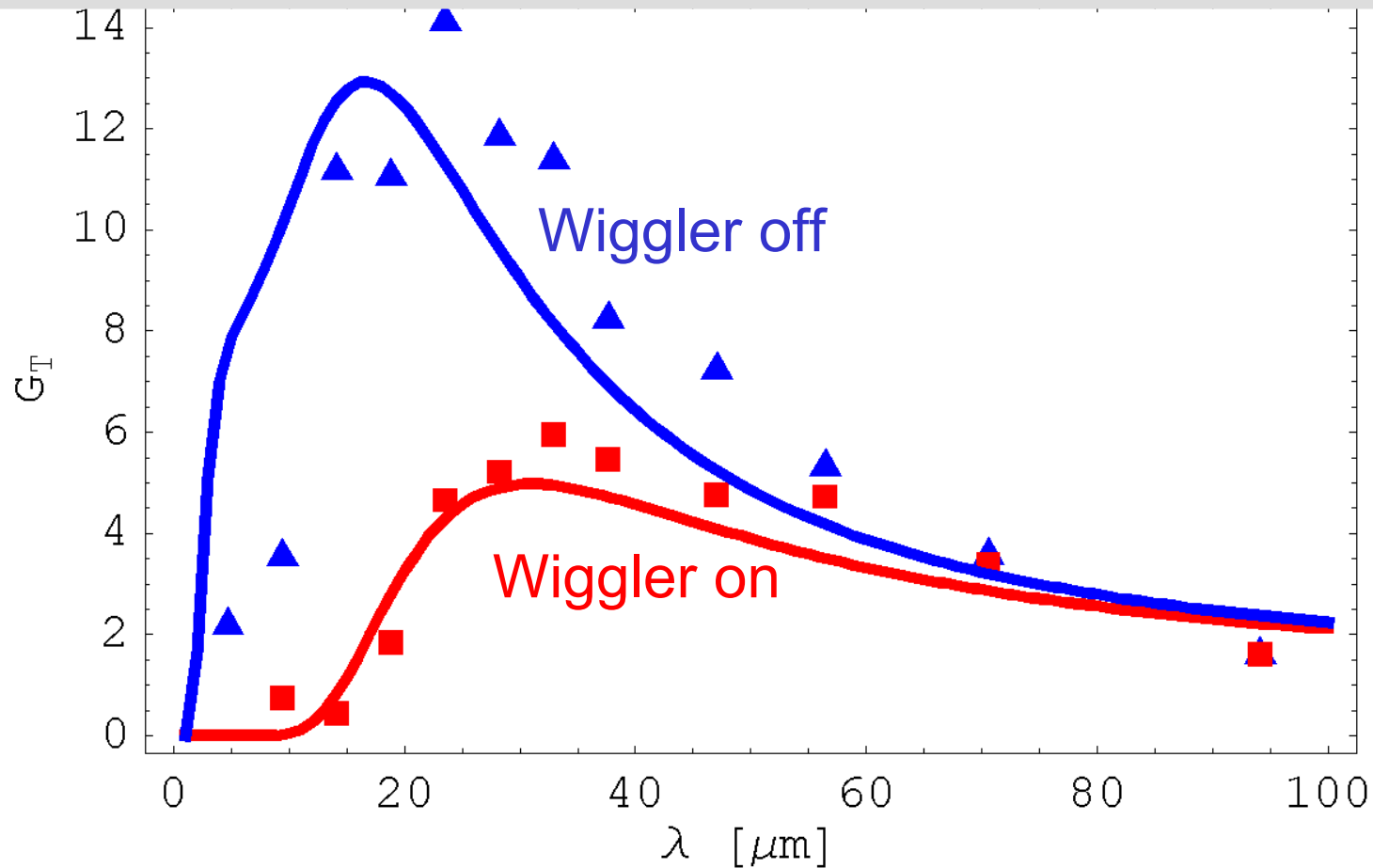
- CSR microbunching can be generated by initial energy modulation of the bunch due to upstream wakefields (such as CSR)

$$b^p = -ikR_{56}(\Delta p)_0 (\exp + \text{one-stage amplification})$$

- Total gain of BC1+BC2 🕒 gain in BC1 X gain in BC2 + energy modulation from BC1 converted to density gain in BC2



BC1+BC2



Reasonable agreement with P. Emma's Simulation



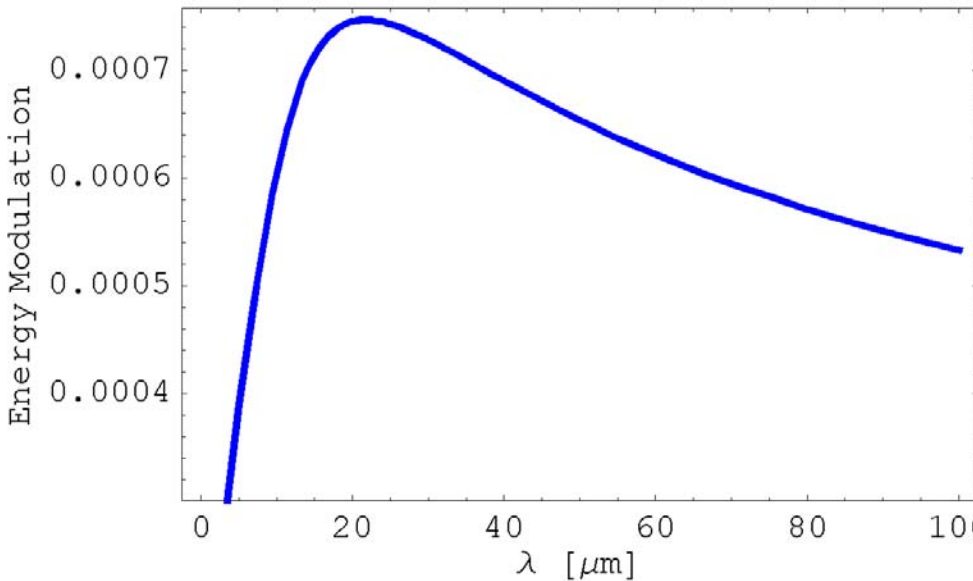
Dog Legs

- Total gain of two chicanes can be large (>10) even though the gain in each chicane is small (<3)
- LCLS has more bends than BCs, Dog Legs (DLs) for beam transport (DL1+DL2)
- DLs typically have very small $R56$ (compared to BCs) → ignore density gain but keep energy modulation (same approximation made within bends of a single chicane)
- DL1 energy modulation can be turned into BC1 density modulation through $R56$ of BC1, cascading through the whole system and leading to more gain in CSR microbunching

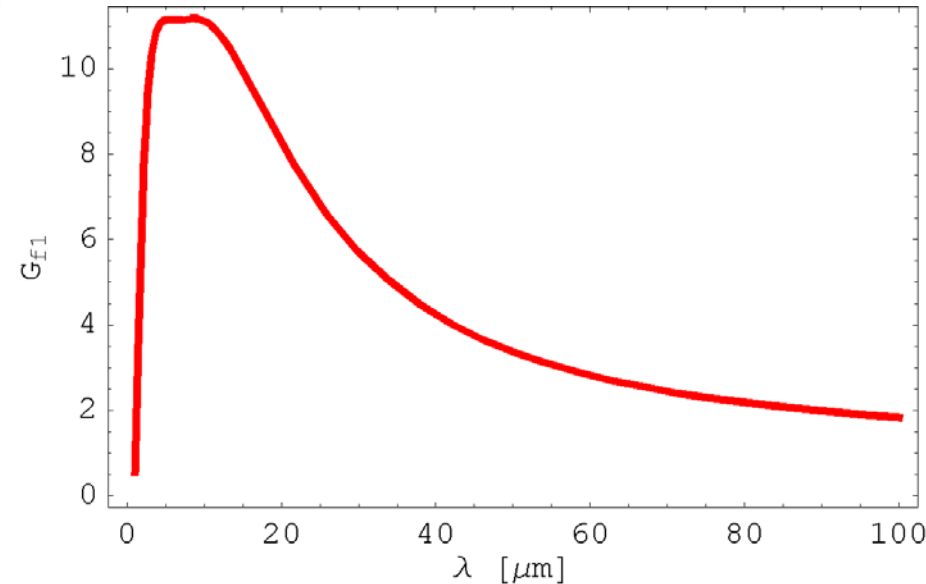


DL1+BC1

- Assume only initial density modulation before DL1



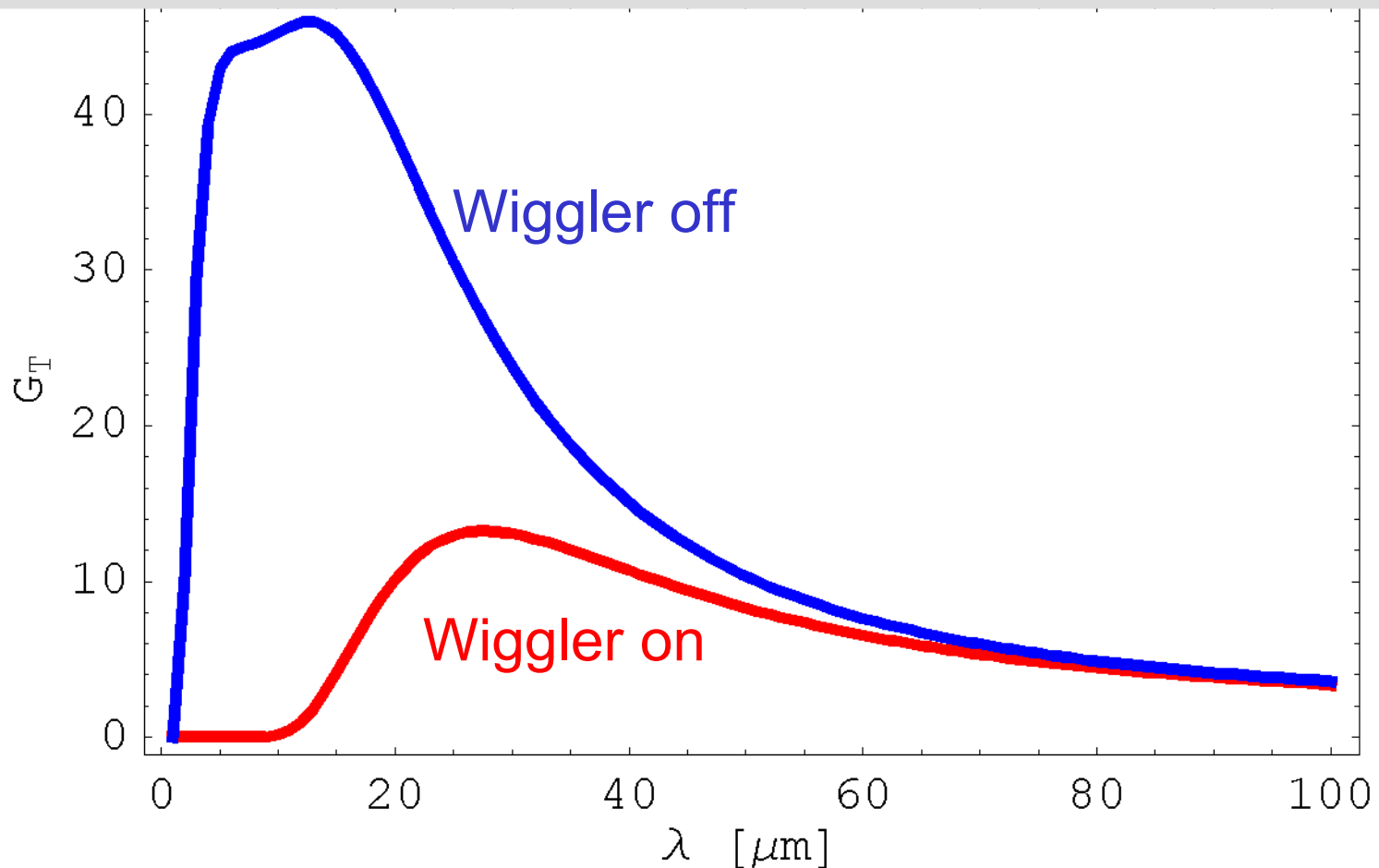
induced energy modulation
at the exit of DL1 (in units
of initial bunching)



Turns into BC1 density gain
through R56 at end of BC1



Estimated Gain of All LCLS Bends



SC wiggler suppresses a short-wavelength, high-gain peak



Comments on Initial Condition

- From shot noise

$$b_{\text{eff}} \sim \frac{1}{\sqrt{N_{\text{coherence length}}}} \sim (10^{-4} \rightarrow 10^{-5})$$

with a gain less than 100, this should be a small effect

- From sharp current spike

→ $b \approx \frac{N_{\text{spike}}}{N_{\text{total}}}$ How big?



- Other sources of energy modulation (wakefields...)
- Watch out for numerical noise in simulations!



Conclusion

- CSR microbunching in a bunch compressor is studied using the iterative solution of the integral equation
- Initiated by density and/or energy modulation
- Cascading effects of multiple chicanes
- Gain curves agree with numerical solution and simulation
- Significant gain is found for the LCLS system (DL1+BC1+BC2+DL2), can be suppressed by increasing the uncorrelated energy spread through a SC wiggler