

## Homework Problems: Lecture 1

1a) Show that the expression  $p=qRB$  is the correct relativistic expression for the motion of a charged particle moving under the influence of a magnetic field.

1b) Show that for an elementary particle of charge  $e$  ( $=1.6 \times 10^{-19}$  Coulombs) the relationship between the particle momentum as measured in MeV/c, the magnetic field as measured in Tesla, and the radius of curvature measured in meter:

$$p(\text{MeV}/c) = 300 \times R(\text{meters}) \times B(\text{Tesla})$$

1c) Estimate the circumference of an accelerator providing 50 TeV ( $1 \text{ TeV} = 10^{12} \text{ eV}$ ) protons with a magnetic bend field of 10 T.

2. In a high energy circular electron accelerator the economics is dominated by synchrotron radiation considerations. Consider a simplified model in which the construction cost is given by three terms:

$$\text{Cost} = \alpha + \beta R + \gamma U$$

where  $\alpha$ ,  $\beta$ , and  $\gamma$  are constants and  $R$  is the radius of the accelerator and  $U$  is the synchrotron radiation energy loss per turn. ( $\alpha$  represents fixed costs that don't depend on the energy or size of the accelerator, for example injection systems or office buildings,  $\beta R$  represents costs that scale linearly with the size of the accelerator, for example the tunnel, and  $\gamma U$  represents costs that scale linearly with the energy loss per turn, for example the rf system.) Show that in this model the optimum radius from the cost perspective is proportional to  $E^2$ , the optimum magnetic field is proportional to  $1/E$ , and that for small fixed costs the optimum cost is proportional to  $E^2$ .

3. Show from fundamental principles (like Maxwell's equations) that it is not possible to accelerate a particle in a circular accelerator using electrostatic fields.