Physics 575 – Accelerator Physics and Technologies for Linear Colliders

Chapter 6 RF Structures (Room Temperature)

Juwen Wang
Stanford Linear Accelerator Center

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6.1 Introduction

6.1.1 Circular or linear

In conventional practice, the mean radius of a circular machine is chosen to be proportional to the square of the energy of the colliding beams to reduce the rapid increase of synchrotron radiation loss and thus to minimize the combined construction and operating cost. Even so, the cost of building a fixed luminosity storage ring scales as the square of the energy of its colliding beams. For example, a circular collider for 500 x 500 GeV is estimated to be 200 km in circumference. The high-energy linear collider is more cost effective because the synchrotron radiation losses are negligible.

6.1.2 Challenges to accelerator structures for linear colliders

There are two most basic requirements for the room temperature accelerating structures for linear colliders:

- The first is to have highly efficient accelerating system with life-time long and stable operation at very high accelerating gradient in order to optimize the machine cost.
- The second is to strongly suppress the long range wakefields for train of bunches in order to achieve very high luminosity for extremely dense beams with very low emittance and small energy spread.

6.1.3 Scope of this chapter

This chapter briefly introduces the most important aspects for the room temperature RF linac structures. They include:

- Historical and background information.
- Basic ideas and fundamental principals for RF linear accelerator structures.
- Practical examples – up-to-date development in NLC R&D.

6.2 Basic

6.2.1 Accelerator systems

The main elements of linear accelerator can be showed schematically by the following figure for SLAC Linear Collider (SLC).

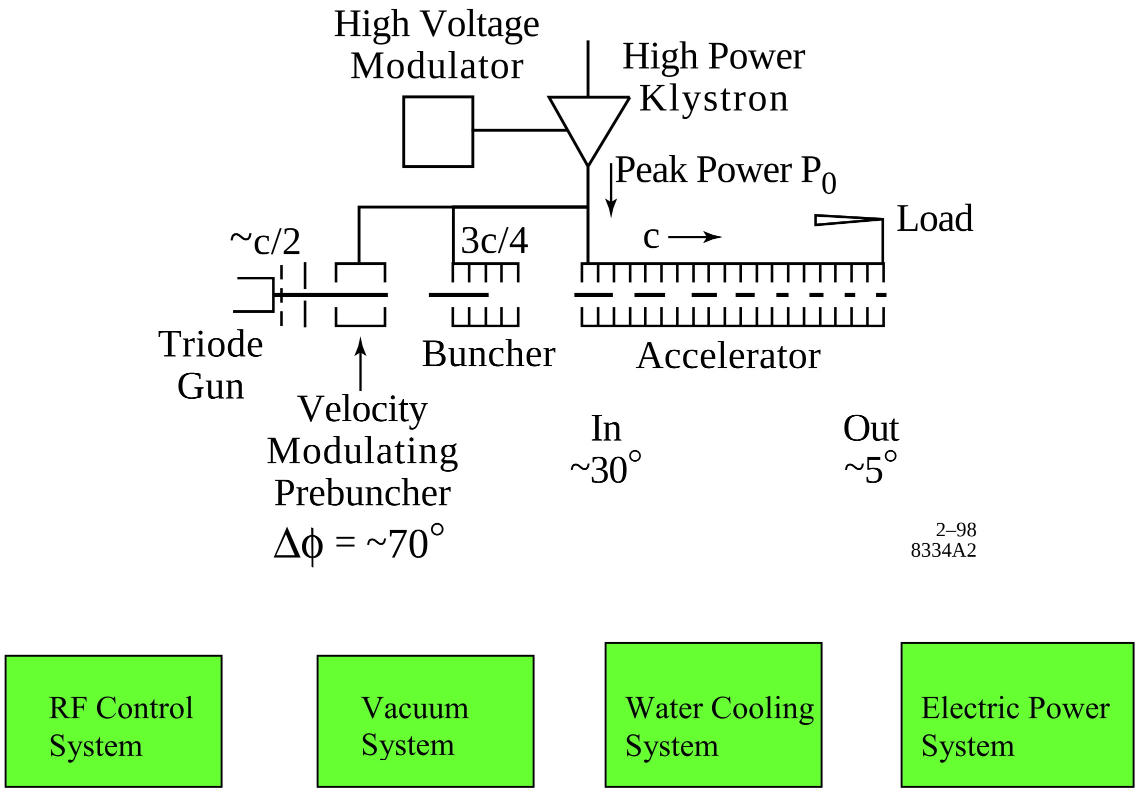


Figure 6.1. Schematic of accelerator systems

6.2.2 History of development for linear accelerator structures

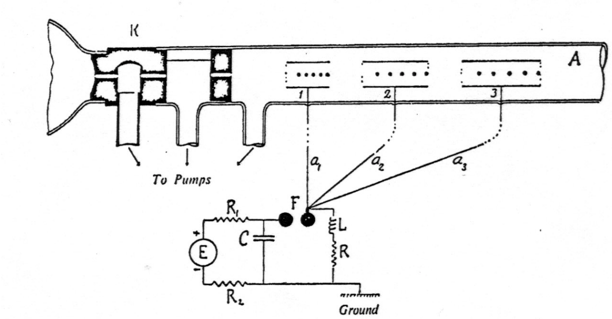
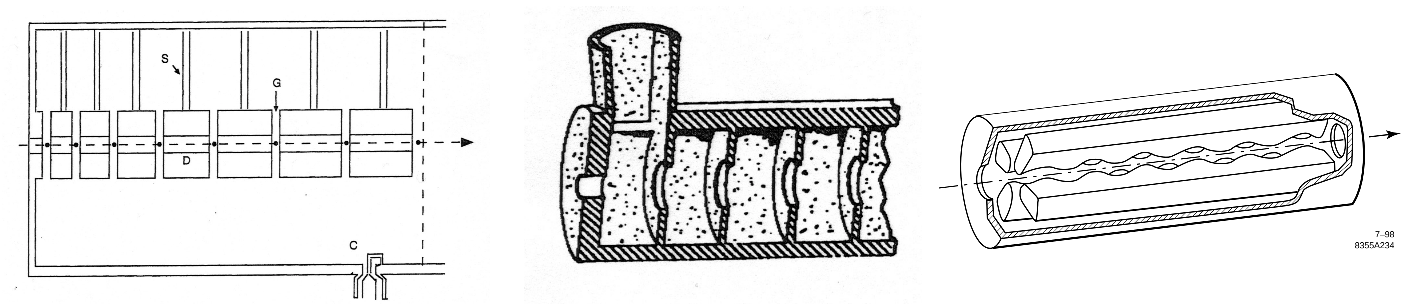


Figure 6.2. Wideröe's linear accelerator and its circuit (25 kV, 1MHz)

The first formal proposal and experimental test for a linac was by Rolf Wideröe in 1928. The linear accelerator for scientific application did not appear until after the development of microwave technology in World War II, stimulated by Radar program.

1955 Luis Alvarez at UC Berkeley, Drift-Tube Linac (DTL).
1947 W. Hansen at Stanford, Disk-loaded waveguide linac.
1970 Radio Frequency Quadruple (RFQ)

The following figures show different structures



Alvarez 200MHz, 32MeV | DLWG | RFQ 6 - 400 MHz 0.01-0.06C

Figure 6.2. Several typical accelerator structures

6.2.3 Modes in Periodic Structure

- Circular Waveguide

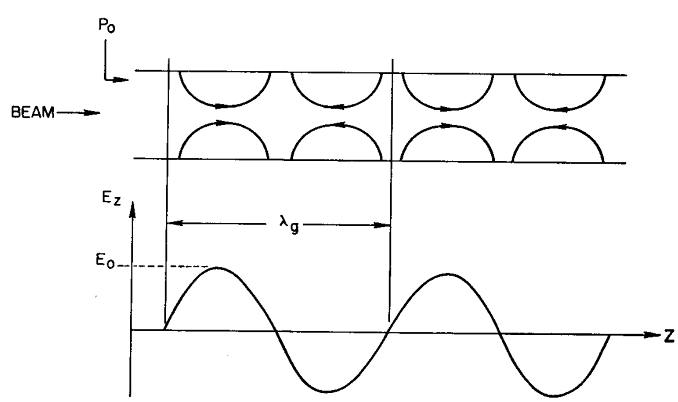


Figure 6.3. TM01 mode pattern and traveling wave axial electric field amplitude in a uniform cylindrical waveguide.

Wave equation for propagation characteristics:

$$\nabla^2 \vec{E} + k^2 \vec{E} = 0$$

$$k = \omega / c$$

and ω is the angular frequency.

For TM $_0$ mode (transverse magnetic field without θ dependence – most simple accelerating mode) in cylindrically symmetric waveguide,

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \left[\left(\frac{\omega}{c}\right)^2 - \beta^2\right] E_z = 0$$

The solution for TM_{01} mode is as follows:

$$E_z = E_0 J_0 (k_c r) e^{j(\omega t - \beta z)}$$
$$E_r = jE_0 \sqrt{1 - (\omega_c / \omega)^2} J_1 (k_c r) e^{j(\omega t - \beta z)}$$
$$H_\theta = j\eta E_0 J_1 (k_c r) e^{j(\omega t - \beta z)}$$

β is propagation constant and η is the intrinsic impedance of the medium, we can consider that k_c and β to be the r and z component of k of plane wave in free space.

$$k_c{}^2 = \left(\frac{\omega}{c}\right)^2 - \beta^2 = \left(\frac{\omega_c}{c}\right)^2$$

For perfect metal boundary condition at wall, $E_z = 0$ (the lowest frequency mode):

$$E_z(b) = 0 \Rightarrow J_0(k_c b) = 0$$
$$k_c b = 2.405$$
$$\omega_c = k_c c = 2.405 c / b$$

where $f_c = 2\pi\omega_c$ is called cut-off frequency.

For any propagation wave, its frequency f must be greater than f_c , the field is in form of $e^{j(\omega t - \beta z)}$ with $\beta > 0$.
For example, diameter 2b = 8 cm tube, f_c = 1.9 GHz, typical S-Band RF (2.856 GHz) can propagate as TM_{01} mode.

- Phase Velocity and Group Velocity

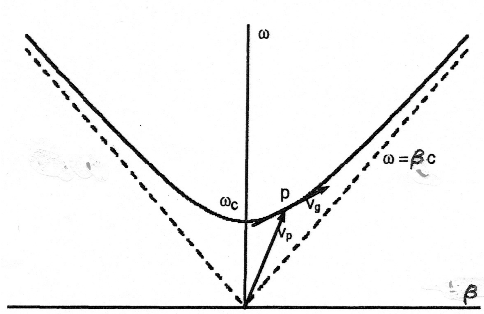


Figure 6.4. ω-β diagram for guided wave in a uniform (unloaded) waveguide.

The phase velocity Vp is the speed of RF field phase along the accelerator, it is given by

$$V_p = \frac{\omega}{\beta}$$

Group velocity is defined as energy propagation velocity.
For wave composed of two components with different frequency ω_1 and ω_2 wave number β_1 and β_2, the wave packet travels with the velocity:

$$V_g = \frac{\omega_1 - \omega_2}{\beta_1 - \beta_2} \to \frac{d\omega}{d\beta}$$

In order to use RF wave to accelerate particle beam, it is necessary to make simple cylinder "loaded" to obtain

$$V_p \le c$$

For uniform waveguide, it is easy to find:

$$V_p V_g = c^2$$

- Coupled Resonator Chain – Periodic Structure

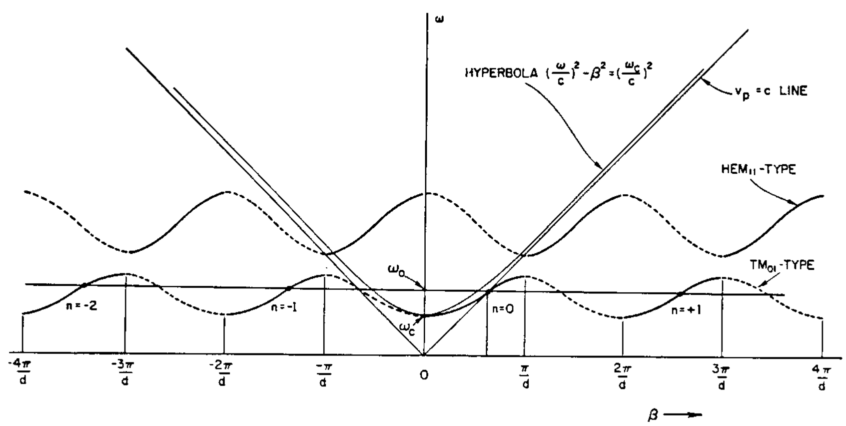


Figure 6.5. Brillouin (ω-β) diagram showing propagation characteristics for uniform and periodically loaded structures with load period d.

Floquet Theorem: When a structure of infinite length is displaced along its axis by one period, it can not be distinguished from original self. For a mode with eigen frequency ω:

$$\vec{E}(\vec{r}, z + d) = e^{-j\beta d}\, \vec{E}(\vec{r}, z) \qquad \vec{r} = x\hat{x} + y\hat{y}$$

where βd is called phase advance per period.
Make Fourier expansion for most common accelerating TM_{01} mode:

$$E_z = \sum_{-\infty}^{\infty} a_n J_0(k_n r) e^{j(\omega t - \beta_n t)}$$

Each term is called space harmonics.

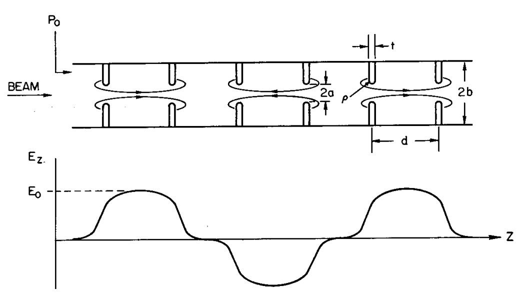
The propagation constant is $$\beta_n = \beta_0 + \frac{2\pi n}{d} = \frac{\omega}{V_{p0}} + \frac{2\pi n}{d}$$

$$k_n^2 = k^2 - \beta_n^2$$

- It is interesting to notice that when the fundamental harmonic n = 0 travels with Vp = c, then $k_0 = 0$, $\beta_0 = k$ and $J_0(0)=1$, the acceleration is independent of the radial position for the synchronized particles.
- Each mode with specific eigenfrequency has unique group velocity.
- Higher order space harmonics does not have contribution to acceleration, but takes RF power.

- RF parameters for accelerating modes

  Mode which is defined as the phase shift per structure period:
  φ = 2π/m where m is the cavity number per wavelength.



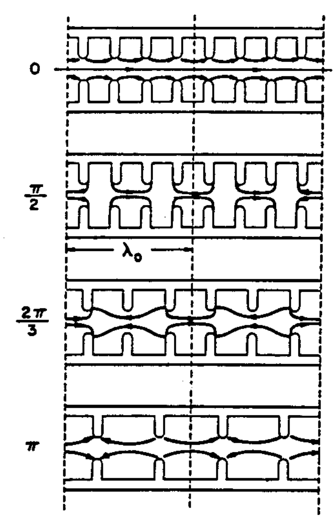


Fig. 6.6. Snapshots of electric field configurations for disk-loaded structures with various phase shift per period (left up for π/2 mode and right for 0, π/2, 2π/3, π mode).
Traveling wave axial electric field amplitude along z-axis for π/2 mode (left lower).

The shunt impedance per unit length r, which measures the accelerating quality of a structure is defined as

$$r = -\frac{E_a^{\ 2}}{dP/dz} \qquad \text{Unit of M}\Omega\text{/m or }\Omega\text{/m}$$

where Ea is the synchronous accelerating field amplitude and dP/dz is the RF power dissipated per unit length.

$$R = \frac{V^2}{P_d} \qquad \text{Unit of M}\Omega\text{ or }\Omega$$

The factor of merit Q, which measures the quality of an RF structure as a resonator.

For traveling wave structure:

$$Q = \frac{-\omega w}{dP / dz}$$

where w is the RF energy stored per unit length and ω is the angular frequency and dP/dz is power dissipated per unit length.

For standing wave structure.

$$Q = \frac{\omega W}{P_d}$$

The group velocity Vg which is the speed of RF energy flow along the accelerator is given by

$$V_g = \frac{P}{w} = \frac{-\omega P}{QdP / dz} = \frac{d\omega}{d\beta}$$

The attenuation factor τ of a constant-impedance or constant-gradient

$$\frac{dE}{dz} = -\alpha E \qquad \frac{dP}{dz} = -2\alpha P$$

α is the attenuation constant in nepers per unit length.

τ for traveling-wave section is defined by the following expression:

$$\frac{P_{out}}{P_{in}} = e^{-2\tau}$$

- For a constant-impedance section, the attenuation is uniform,

$$\alpha = \frac{-dP/dz}{2P} = \frac{\omega}{2V_g Q} \qquad \tau = \alpha L = \frac{\omega L}{2V_g Q}$$

where L is the length of the section.

- For non-uniform structures,

$$\tau = \int_0^L \alpha(z)dz$$

- For a constant-gradient section, the attenuation constant α is a function of z: $\alpha = \alpha(z) = \omega / 2V_g(z)Q$

We have the following expression

$$dP/dz = -2\alpha(z)P = const = \frac{P_{in}(1-e^{-2\tau})}{L}$$

r/Q ratio this is a important factor depending on geometry measures the accelerating field for certain stored energy, which is independent with material, machining quality.

$$\frac{r}{Q} = \frac{E^2}{\omega w}$$

The filling time t_F

For traveling wave structure, the field builds up "in Space". The filling time is the time needed to fill the whole section of either constant impedance or constant gradient, which is given by

$$t_F = \int_0^L \frac{dz}{V_g} = \frac{Q}{\omega}\int_0^L \frac{-dp/dz}{P} dz = \frac{2Q}{\omega}\tau$$

The field in SW structures builds up "in Time". The filling time is defined as the time needed to build up the field to (1-1/ e) = 0.632 times the steady-state field:

$$t_F = \frac{2Q_L}{\omega} = \frac{2Q_0}{(1+\beta_c)\omega} \qquad \beta_c = \frac{Q_0}{Q_{ex}}$$

where Q_0 is the unloaded Q value, $Q_0 = \omega w/P_d$,
Q_{ex} is external Q value, $Q_{ex} = \omega w/P_{ex} = Q_0/\beta$,
Q_L is loaded Q value, $Q_L = \omega w/(P_d+P_{ex}) = Q_0/(1+\beta)$,
β_c is the coupling coefficient of the section to the input microwave network.

The Working Frequency is a first and important parameter to choose in accelerator design. Almost all basic RF parameters have frequency dependence.

$$r \propto \sqrt{f} \qquad size \propto \frac{1}{f} \qquad Q \propto 1/\sqrt{f} \qquad \frac{r}{Q} \propto f$$

- Structure types

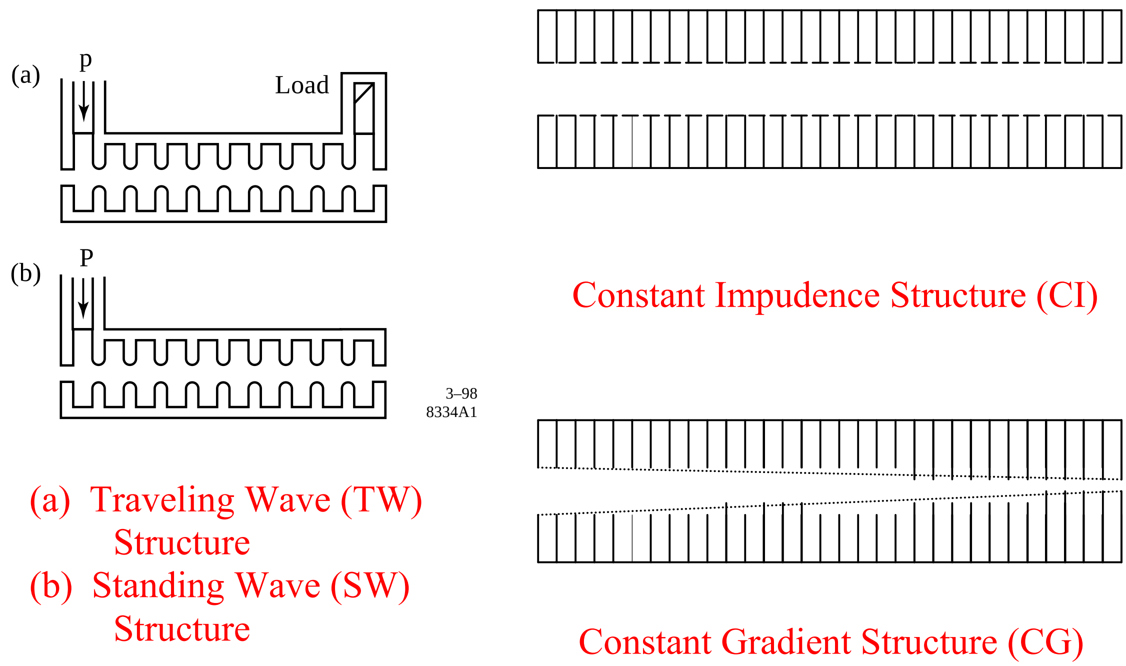


Figure 6.7. Schematic of disk-loaded TW and SW structures.

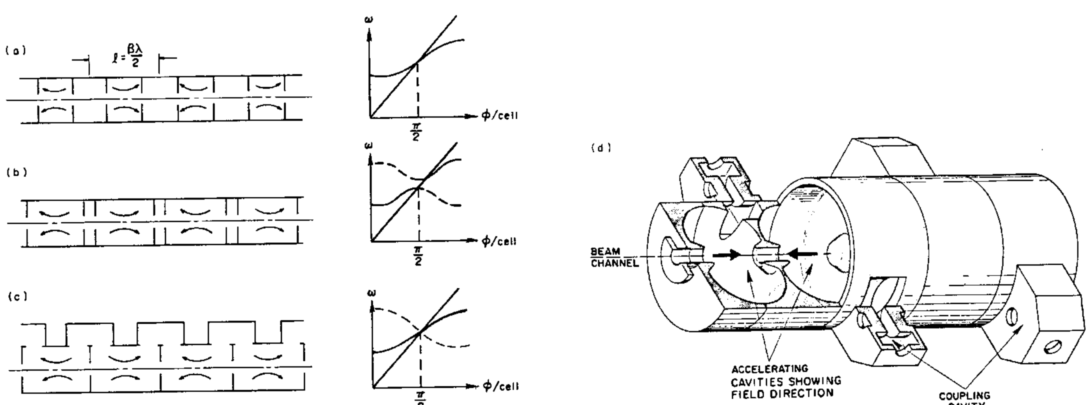


Figure 6.8. Evolution of π/2 mode disk-loaded structure to π mode side coupled structure.

6.2.4 Longitudinal Dynamics

Energy of particle: $u = \frac{m_0 c^2}{\sqrt{1-\beta_e^2}}$

Energy change with time: $\frac{du}{dt} = -eE_z \frac{dz}{dt} Sin\theta$

$\theta = \frac{\omega z}{V_p} - \omega t_e$ where t is the time of particle reaches to z

$\theta = \omega \int (\frac{1}{V_p} - \frac{1}{V_e}) dz$

$d\theta = \omega(\frac{1}{V_p} - \frac{1}{V_e}) dz$ when particle is faster (Ve>Vp), dθ>0 and vise versa

It is convenient to use z as variable and $\frac{d}{dt} = \frac{dz}{dt}\frac{d}{dz}$

The longitudinal motion is described by the following two equations:

$$\frac{du}{dz} = -eE_z Sin\theta$$

$$\frac{d\theta}{dz} = \frac{2\pi}{\lambda}\left(\frac{1}{\beta_p} - \frac{u}{\sqrt{u^2 - u_0^2}}\right)$$

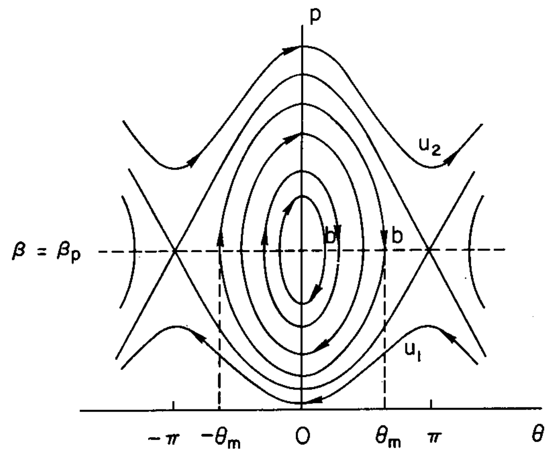
The simplify the equations of motion, the normalized momentum is

$$p = \frac{mV_e}{m_0 c} = \gamma\beta_e = \sqrt{\gamma^2 - 1}$$

Integration using variable substitution of $\gamma^2 = p^2 + 1$,

The equation for the orbit in phase space is

$$\cos\theta - \cos\theta_m = \frac{2\pi m_0 c^2}{eE_0 \lambda}\left[\sqrt{p^2+1} - \sqrt{1-\beta_p^2} - \beta_p p\right]$$



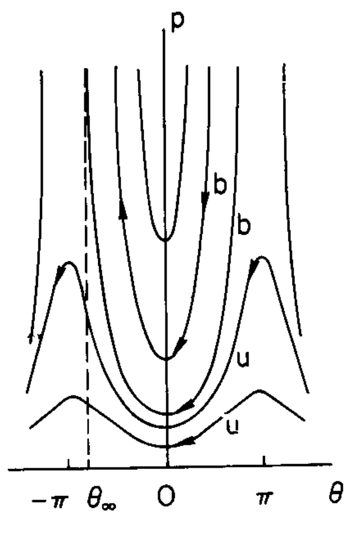


Figure 6.9. Longitudinal phase space for β_p<1.

Figure 6.10. Longitudinal phase space for β_p=1.

- Phase velocity less than c (β_p<1)

When 1>cosθ>-1 the particles oscillate in p and θ plan with elliptical orbits centered around ($\beta = \beta_p$, $\theta = 0$). If an assembly of particles with a relative large phase extent and small momentum extent enters such a structure, then after traversing ¼ of a phase oscillation it will have a small phase extent and large momentum extent, we call this action as bunching.

- Phase velocity equals c (β_p=1)

When β_p=1, dθ/dz is always negative, and the orbits become open-ended as shown in Figure 6.10. The orbit equation becomes

$$\cos\theta - \cos\theta_m = \frac{2\pi m_0 c^2}{eE_0\lambda}\left[\sqrt{p^2+1} - p\right] = \frac{2\pi m_0 c^2}{eE_0\lambda}\sqrt{\frac{1-\beta_e}{1+\beta_e}}$$

where θ_m has been renamed θ_∞ to emphasize that it corresponds to p = ∞. The threshold accelerating gradient for capture is $\cos\theta - \cos\theta_\infty = 2$, or

$$E_0(threshold) = \frac{\pi m_0 c^2}{e\lambda}\left[\sqrt{p_0^{\,2}+1} - p_0\right]$$

Let us discuss an interesting case: a particle entering the structure with a phase θ_0=0, has an asymptotic phase θ_∞= $-\pi/2$, thus the an assembly of particles will get maximum acceleration and maximum phase compression. For small phase extents $\pm\Delta\theta_0$ around θ_0=0,

$$\theta_\infty \approx -\frac{\pi}{2} - \frac{(\Delta\theta_0)^2}{2}$$

Let us consider a practical example at SLAC (λ=10.5 cm). If 80 KeV electrons enter an accelerator section with optimized accelerating gradient (home work) and have the above idea bunching, a 30^O bunch ends up in less a 2^O bunch.

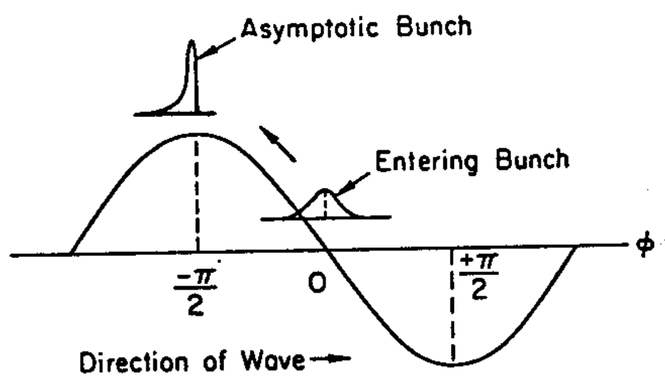


Figure 6.11. Asymptotic bunching process in Vp=c constant-gradient accelerator section with value of accelerating gradient E optimized for entrance condition.

- Acceleration for Traveling Wave (TW) structures

Constant Impedance (CI) | Constant Gradient (CG)

1. The energy gain V of a charged particle is given by

CI: $$V = \sqrt{2\tau}\left[(1-e^{-\tau})/\tau\right]\sqrt{P_{in} rL}$$ CG: $$V = \sqrt{1-e^{-2\tau}}\sqrt{P_{in} rL} = \sqrt{P_{dis} rL}$$

2. The RF energy supplied in the time period t_F can be derived from above:

CI: $$P_{in} t_F = \left(\frac{\tau}{1-e^{-\tau}}\right)^2 \frac{V^2}{\omega \frac{r}{Q} L}$$ CG: $$P_{in} t_F = \left(\frac{2\tau}{1-e^{-2\tau}}\right)^2 \frac{V^2}{\omega \frac{r}{Q} L}$$

3. The energy W stored in the entire section at the end of the filling time is

CI: $$W = \frac{Q}{\omega}\int_0^l \frac{dp}{dz} dz = P_{in}\frac{Q}{\omega}(1-e^{-2\tau})$$ CG: $$W = \int_0^l \frac{P}{v_g} dz = P_{in}\frac{Q}{\omega}(1-e^{-2\tau})$$

- Acceleration for Standing Wave (SW) structures

Due to multi reflection, the equivalent input power is increased:

$$P = P_s + P_s e^{-2\alpha L} + P_s e^{-4\alpha L} + \ldots\ldots = \frac{P_s}{1-e^{-4\alpha L}}$$

The slightly higher energy gain for SW is paid by field building up time (choice of length of SW structures).

For resonant cavity, power feed is related to the RF coupling:

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ex}} \qquad \beta_c = \frac{Q_0}{Q_{ex}}$$

As defined earlier, Q_L, $Q_0 = \omega w/Pd$, $Q_{ex} = \omega w/P_{ex}$ are loaded Q value, cavity Q value, and external Q value.

B_c is coupling coefficient between waveguide and structure.

The energy gain of a charged particle is given:

$$V = (1-e^{-t/t_F})\frac{2\sqrt{\beta_c}}{1+\beta_c}\sqrt{P_{in} rL} = (1-e^{-t/t_F})\sqrt{P_{dis} rL}$$

6.2.6 Beam Loading

The effect of the beam on the accelerating field is called BEAM LOADING. The superposition of the accelerating field established by external generator and the beam-induced field needs to be studied carefully in order to obtain the net Phase and Amplitude of acceleration.

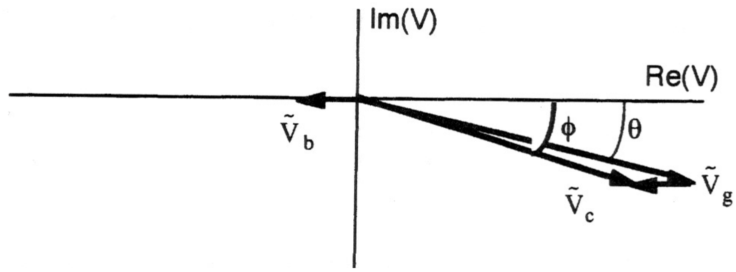


Figure. 6.12. Steady-state Phasors in a complex plane for beam loaded structure: V_g for generator-induced voltage
V_b beam-induced voltage
V_c net cavity voltage

In order to obtain a basic physics picture, we will assume the synchronized bunches in a bunch train stay in the peak of RF field for both TW and SW analysis.

The RF power loss per unit length is given by:

$$\frac{dP}{dz} = \left(\frac{dP}{dz}\right)_{wall} + \left(\frac{dP}{dz}\right)_{beam}$$

Where I is average current, E is the amplitude of synchronized field.

$$E^2 = 2\alpha rP$$

$$E\frac{dE}{dz} = rP\frac{d\alpha}{dz} + \alpha r\frac{dP}{dz} = r\frac{E^2}{2\alpha r}\frac{d\alpha}{dz} - \alpha r\left(\frac{E^2}{r} + EI\right)$$

$$\frac{dE}{dz} = -\alpha E\left(1 - \frac{1}{2\alpha^2}\frac{d\alpha}{dz}\right) - \alpha rI$$

For constant impedance structure: $$\frac{dE}{dz} = -\alpha E - \alpha rI$$

$$E(z) = E(0)e^{-\alpha z} - Ir(1 - e^{-\alpha z}) \qquad E(0) = \sqrt{2\alpha rP_{in}}$$

The total energy gain through a length L is

$$V = \int_0^L E(z)dz = \sqrt{2P_0 rL}\,\frac{1-e^{-\tau}}{\sqrt{\tau}} - IrL\,(1 - \frac{1-e^{-\tau}}{e^{-\tau}})$$

Where P_0 is input RF power in MW, r is shunt impedance per unit length in MΩ/m, L is structure length in m, I is average beam current in Ampere, V is total energy gain in MV.
The first term is unloaded energy gain, and loaded energy decreases linearly With the beam current.

For constant gradient structures: $\frac{dE}{dz} = -\alpha r I$

$$E = E_0 + \frac{rI}{2}\ln\left(1 - \frac{z}{L}(1 - e^{-2\tau})\right)$$

The attenuation coefficient is

$$\alpha(z) = \frac{(1-e^{-2\tau})/2L}{1-(1-e^{-2\tau})(z/L)}$$

The complete solution including transient can be expressed as

$$t_F \le t \le 2t_F \qquad V(t) = E_0 L + \frac{rI}{2}\left[\frac{\omega L e^{-2\tau}}{Q(1-e^{-2\tau})}t - \frac{L}{1-e^{-2\tau}}(1 - e^{-\frac{\omega}{Q}t})\right]$$

$$t \ge 2t_F \qquad V(t) = E_0 L - \frac{rIL}{2}\left[1 - \frac{2\tau e^{-2\tau}}{1-e^{-2\tau}}\right]$$

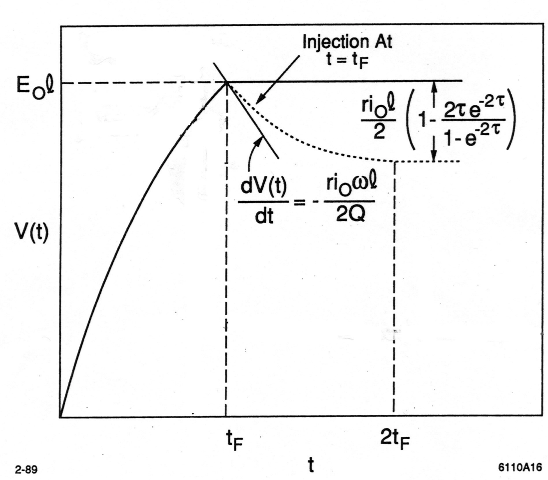


Figure 6.13. Transient beam loading in a TW constant gradient structure.

For a standing wave structure with a Coupling coefficient β_c,
The energy gain V(t) is

$$V(t) = (1 - e^{-t/t_F})\frac{2\sqrt{\beta_c}}{1+\beta_c}\sqrt{P_{in} rL} = (1 - e^{-t/t_F})\sqrt{P_{dis} rL}$$

$$V(t) = (1 - e^{-t/t_F})\sqrt{P_{dis} rL} \;\; - \frac{irL}{1+\beta}(1 - e^{-\frac{t-t_b}{t_F}})$$

If the beam is injected at time t_b and the coupling coefficient meets the following condition:

$$\beta_c = 1 + \frac{P_b}{P_{dis}}$$

We will have;

$$P_{in} = P_{dis} + P_b$$

There is no reflection from the structure to power source with beam. From above formula, the beam injection time is

$$t_b = -t_F \ln(1 - \frac{V_b}{V_0}) = t_F \ln\frac{2\beta_c}{\beta_c - 1}$$

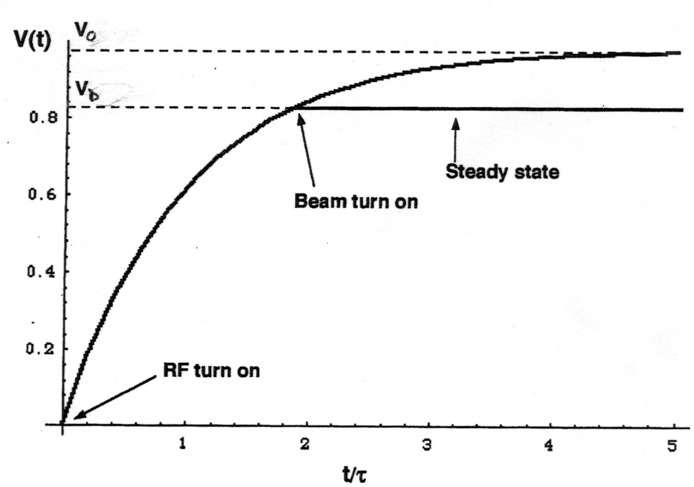


Figure 6.14. Transient beam loading in a standing wave structure.

6.2.7 Wakefields

The wakefield is the scattered electromagnetic radiation created by relativistic moving charged particles in RF cavities, vacuum bellows, and other beam line components.

These fields effect on the particles themselves and subsequent charged particles.

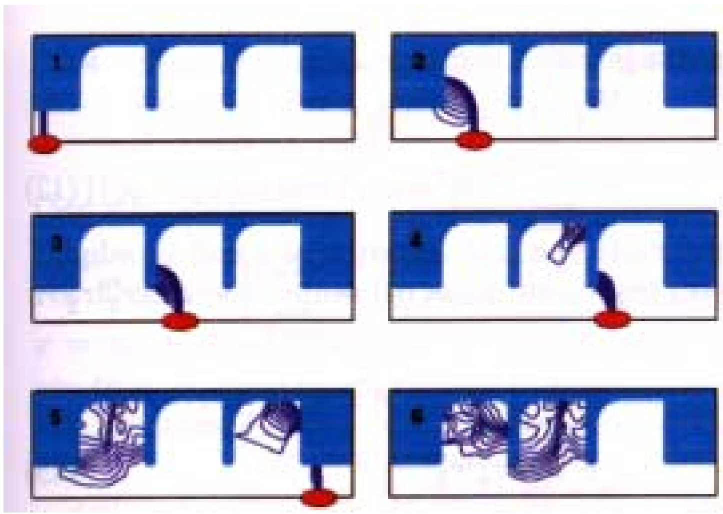


Figure 6.15. Electric field lines of a bunch traversing through a three-cell disk-loaded structure.

- No disturbance ahead of moving charge ----- CAUSALTY.
- Wakefields behind of the moving charge vary in a complex way – in space and time.
- The fields can be decomposed into MODES.
- Each mode has its particular FIELD PATTERN and oscillates with its own eigenfrequency.
- For simplified analysis, the modes are orthogonal, i.e. the energy contained in a particular mode does not has energy exchange with other modes.

Let's consider a point charge with charge Q moving at the speed of light along a path in z direction through a discontinuity L:

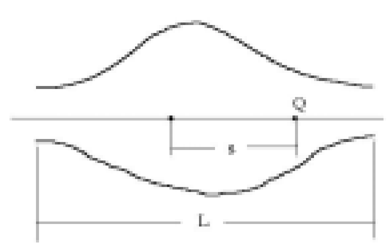


Figure 6.16. Notations for a point charge traversing through a discontinuity.

- Longitudinal wakefields

For practical interest, the bunches (both driving charged particle and test charged particle) are near the structure axis.
We define the longitudinal delta-function potential Wz(s) as the potential (in Volt/Coulomb) experienced by the test particle following along the same path at time τ (distance s= τc) behind of the unit driving charge.

$$W_z(s) = -\frac{1}{Q}\int_0^L dz E_z(z, \frac{z+s}{c})$$

The longitudinal wakefields are dominated by the m=0 modes, for example, TM_{01}, TM_{02},….

$$W_z(s) = \sum_n k_n \cos(\frac{\omega_n s}{c}) \times \begin{matrix} 0(s<0) \\ 1(s=0) \\ 2(s>0) \end{matrix}$$

The loss factor k_n: $$k_n = \frac{|V_n|^2}{4U_n} = \frac{\omega_n}{4}(\frac{R_n}{Q_n})$$

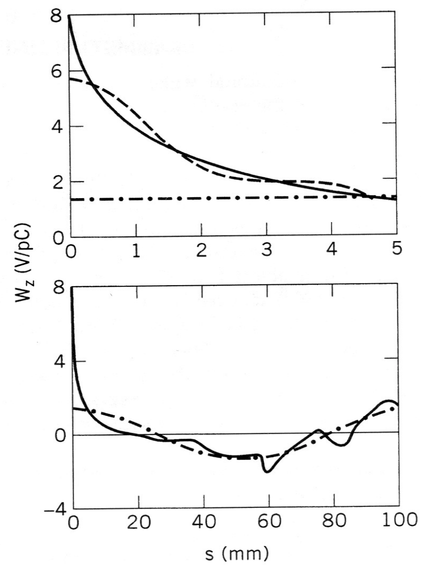
where Un is the stored energy for nth mode.
Vn is the maximum voltage gain from nth mode for a unit test particle with speed of light.
The total amount of energy deposited in all the modes by the driving charge:

$$U = Q^2 \sum_n k_n$$

Longitudinal wakefields are approximately independent of the transverse position of both the driving charges and testing charges.

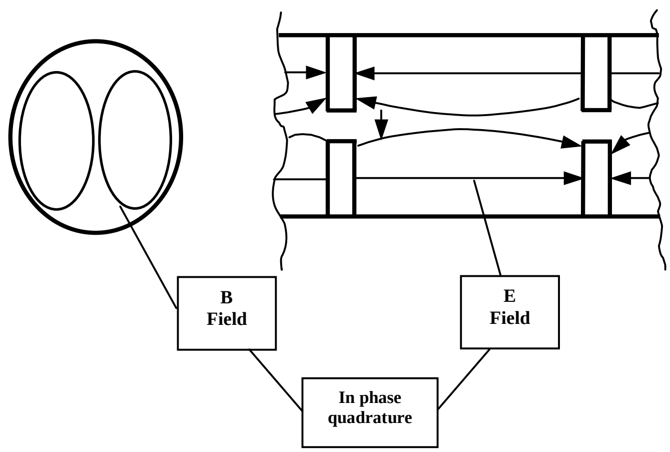
Short range longitudinal wakefield --- Energy spread within a bunch.
Long range longitudinal wakefield --- Beam loading effect.

Figure 6.17. Computed longitudinal δ-function wake potential per cell for S-Band SLAC structure:
Solid line: Total wake
Dashed line: 450 modes
Dot-dashed line: Accelerating mode (λ=105 mm)



- Transverse wakefields

Figure 6.18. As a example, Schematic of field Pattern for the lowest frequency mode -- HEM_{11} Mode.



The transverse wake potential is defined as the transverse momentum kick experienced by a unit test charge following at a distance s behind on the same path with a speed of light.

$$W_\perp = \frac{1}{Q}\int_0^L dz \left[\vec{E}_\perp + (\vec{v} \times \vec{B})_\perp \right]_{t=\frac{z+s}{c}}$$

The transverse wakefields are dominated by the dipole modes (m>1), For example, HEM_{11}, HEM_{21},…

Approximately, we can have: $$W_\perp \approx \left(\frac{r'}{a}\right)\hat{x}\sum_n \frac{2k_{1n}}{\omega_{1n} a / c} Sin\left(\frac{\omega_{1n} s}{c}\right) \qquad s \geq 0$$

where r′ is the transverse offset of the driving charge and the charge is on x axis.

a is the tube radius of the structure.

k_{1n} for m=1 nth dipole mode has similar definition like m=0 case.

The unit of transverse potential V/Coulomb/mm.

The transverse wakefields depend on the driving charge as the first power of its offset r', the direction of the transverse wake potential vector is decided by the position of the driving charge and independent with the test charge position.

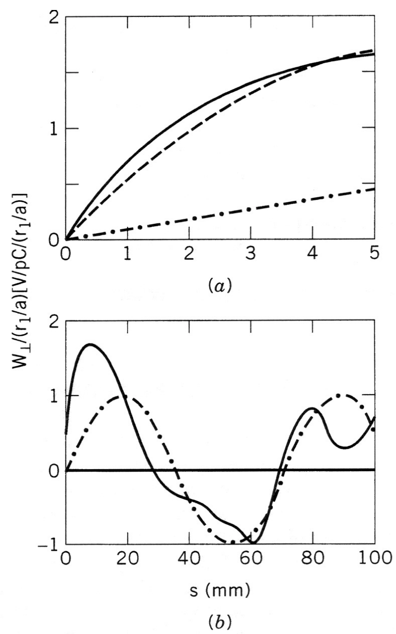


Figure 6.18. Computed transverse δ-function wake potential per cell for S-Band SLAC structure.
Solid line: Total wake
Dashed line: 495 modes
Dot-dashed line: lowest frequency dipole mode (λ=70 mm)

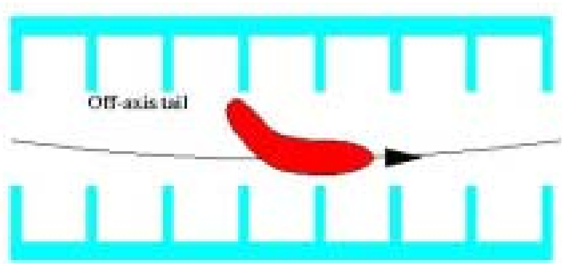


Figure 6.19. Single Bunch Emittance Growth (Head-Tail Instability) Due to Short range transverse wakefields.

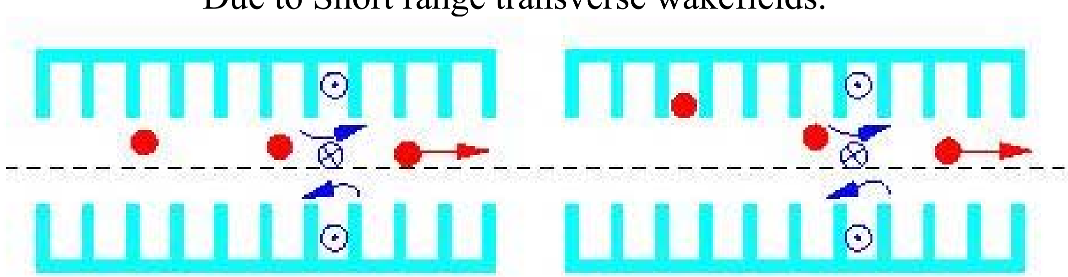


Figure 6.20. Multi-bunch Beam Breakup due to Long range transverse wakefields