Homework Set #7
(due: Wednesday, March 5, 2003)

1. Properties of the SLC damping rings are:

Tunes	$Q_x \approx 8.3$, $Q_y \approx 3.2$
RF Frequency	714 MHz
Harmonic Number	84
Peak RF Voltage	1 MV
Beam Energy	1.20 GeV
Energy Loss per Turn	80 keV

   a. Calculate the synchrotron tune.

   b. Draw the rf bucket in longitudinal phase space (ϕ, $\Delta E/E$).

   c. What is the maximum incoming momentum spread that can be captured?

   d. The incoming beam usually has a 1% rms energy spread and a 1.5 mm rms bunch length. What is the longitudinal emittance (area in ($\delta\phi$, δE) phase space) before and after filamentation?

2. Prove Lionville's theorem in the following steps.

   a. Let the time evolution of the phase space coordinate be given by

$$\frac{dq}{dt} = f(q,p), \quad \frac{dp}{dt} = g(q,p)$$

   Consider a small rectangular phase space area bounded by the points A, B, C, D (see figure). After an infinitesimal time dt, the points evolve to (A′, B′, C′, D′). Show that the area of phase space $\Delta q \Delta p$ of the initial square changes to

$$\text{Area of } A'\,B'\,C'\,D' = \Delta q \Delta p \left(1 + \left(\frac{\partial f}{\partial q} + \frac{\partial g}{\partial f}\right) dt\right)$$

b. Show that the phase space area is invariant for a Hamiltonian system.